Dalhousie Distributed Research Institute and Virtual Environment

MAA Online — The Mathematical Association of America

MAA Short Course on Experimental Mathematics (San Antonio Jan 10-11, 2006)

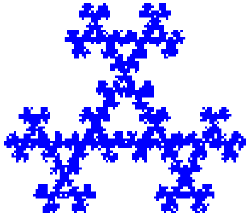
Jonathan Borwein, FRSC www.cs.dal.ca/~jborwein

Canada Research Chair in Collaborative Technology

``intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication.

Therefore, I think that in teaching high school age youngsters we should emphasize intuitive insight more than, and long before, deductive reasoning."

George Polya



DALHOUSIE UNIVERSITY *Inspiring Minds*

Revised 05/01/2006

Excursions in Experimental Mathematics

Welcome and Introductions.

Goal. During these two days we hope to initiate a conversation about how mathematics is done and what it is.

Schedule. On next slide.

Reception. 5.30—6.30 in Terrace Room at Marriot.

Tuesday, January 10

AMS Short Course on Modeling and Simulation of Biological Networks, I

8:00 AM – 5:00 PM

Organizer: **Reinhard Laubenbacher**, Virginia Bioinformatics Institute

8:00AM Registration.

9:00AM *Introduction to the Short Course topics.*
(1) **Reinhard Laubenbacher**, Virginia Polytechnic Inst & State Univ

9:30AM *Reconstructing ancestral genomes.*
(2) **Lior Pachter**, University of California Berkeley

10:45AM Break.

11:15AM *Phylogenetics.*
(3) **Elizabeth Allman**, University of Southern Maine

2:00PM *Modeling and simulation of biochemical networks.*
(4) **Pedro Mendes**, Virginia Bioinformatics Institute

3:15PM Break.

3:30PM Panel Discussion: *The new face of computational biology.*

MAA Short Course: Experimental Mathematics in Action, I

9:00 AM – 5:00 PM

Organizer: **Jonathan M. Borwein**, Dalhousie University

8:00AM Registration.

9:00AM *What is experimental mathematics?*
(5) **Jonathan M. Borwein**, Dalhousie University

10:15AM Break.

10:45AM *Case Study I: Integrals and series using Mathematica.*
(6) **Victor H. Moll**, Tulane University

2:00PM *Algorithms for experimental mathematics, I.*
(7) **David H. Bailey**, Lawrence Berkeley National Laboratory

3:15PM Break.

3:45PM *Case Study II: Discrete math and number theory in Maple and C++.*
(8) **Neil J. Calkin**, Clemson University

Wednesday, January 11

MAA Board of Governors

8:00 AM – 5:00 PM

AMS Short Course on Modeling and Simulation of Biological Networks, II

9:00 AM – 5:00 PM

Organizer: **Reinhard Laubenbacher**, Virginia Bioinformatics Institute

9:00AM *A computational algebra approach to systems biology.*
(9) **Brandilyn Stigler**, Virginia Polytechnic Institute & State Univ

10:15AM Break.

10:45AM *Interaction-based computing approach to modeling and simulations of large biological and socio-technical systems.*
(10) **Madhav Marathe**, Virginia Bioinformatics Institute

2:00PM *Optimal control of population and disease models.*
(11) **Suzanne M. Lenhart**, University of Tennessee

3:15PM Break.

3:30PM Panel Discussion: *Opportunities in computational biology.*

MAA Short Course: Experimental Mathematics in Action, II

9:00 AM – 5:00 PM

Organizer: **Jonathan M. Borwein**, Dalhousie University

9:00AM *Case Study III: Inverse scattering on Matlab.*
(12) **D. Russell Luke**, University of Delaware

10:15AM Break.

10:45AM *Case Study IV: Analysis and probability on the computer.*
(13) **Roland Girgensohn**, Bundeswehr Medical Office

2:00PM *Algorithms for experimental mathematics, II.*
(14) **David H. Bailey**, Lawrence Berkeley National Laboratory

3:15PM Break.

3:45PM *Concluding examples. Putting everything together.*
(15) **Jonathan M. Borwein**, Dalhousie University

That's Mathematics

Tune - "That's Entertainment

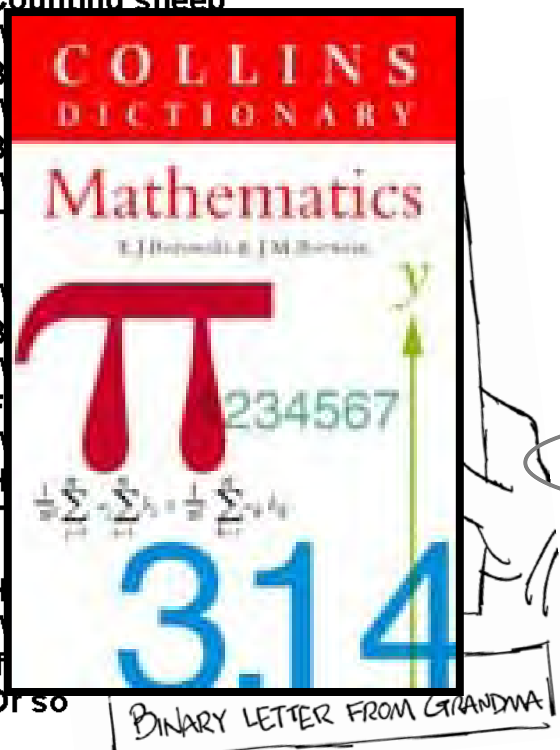
This was most likely written for the Ju
Fermat Fest held in San Francisco.

Counting sheep
W
B
W
B
W
W
T

W
B
W
F
W
H
T

H
W
If
Or so

When y... choose
How much postage to use,
When you know
What's the chance it will snow



Tom Lehrer's last student

July 16, 2002

Jonathan Borwein
CoLab, Dept. of Mathematics
Simon Fraser University
8888 University Drive
Burnaby, BC
Canada V5A1S6

Dear Jonathan Borwein:

As sole copyright owner of the song THAT'S MATHEMATICS, I grant you permission to use it on the CD mentioned in your letter of July 7th, in the manner described therein.

I dare say I should charge a fee for this use, but since I assume the song is already out there on the web in mp3 form without my permission (as many of them are), I can't justify penalizing you for being honest. In other words, there will be no charge.

Geek Heaven!

I own all the rights, by the way, so you don't have to clear it with Rhino. If there is to be any printed material accompanying the CD, the credit should read: © 1995 Tom Lehrer. U

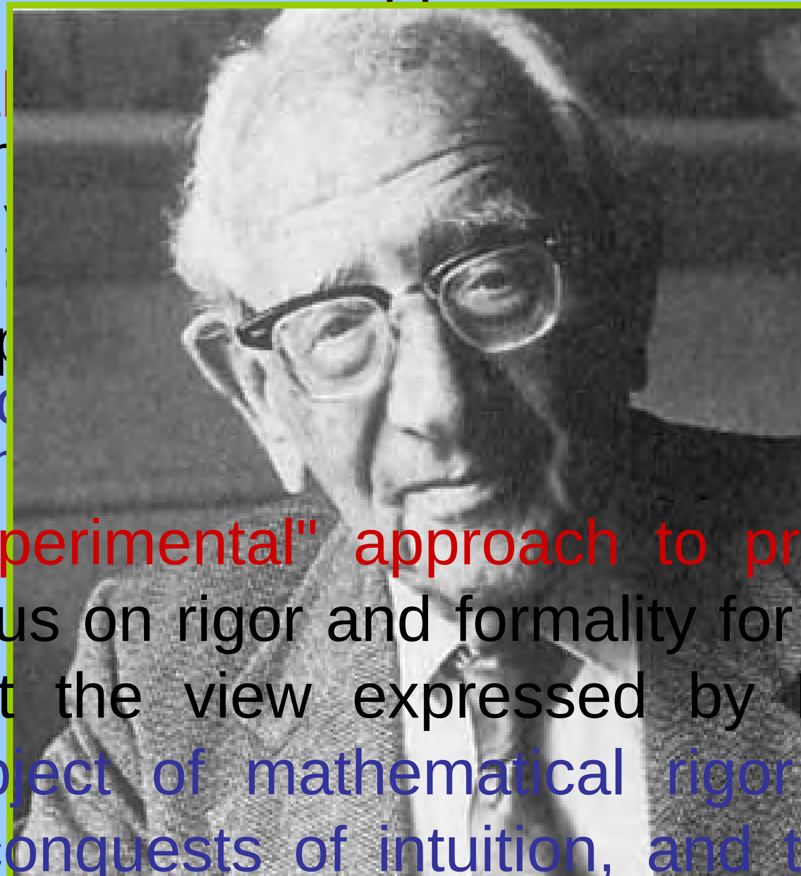
Good luck with your project. ... graduates who
can't determine the remaind... e decimal that
appears on the calculator), I ... ation.

Sincerely yours,

Tom Lehrer

Polya Made Plausible by Computers

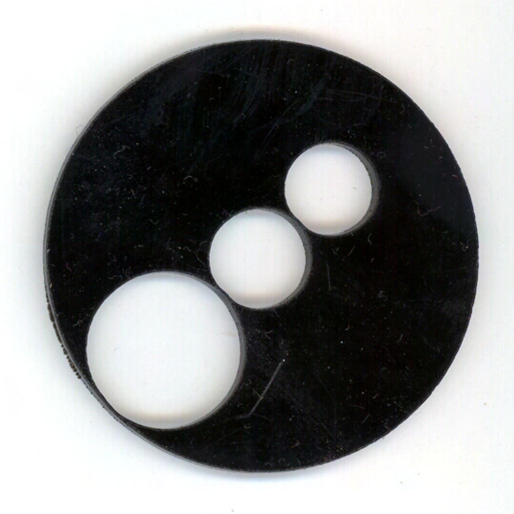
“A mathematical deduction appears to Descartes as a chain of
conclusions, … e steps. What is
needed for t… intuitive insight at
each step wh… on attained by that
step evidentl… lows from formerly
acquired kn… y by intuition or
indirectly by p… hat in teaching high
school age y… size intuitive insight
more than, a… soning.”



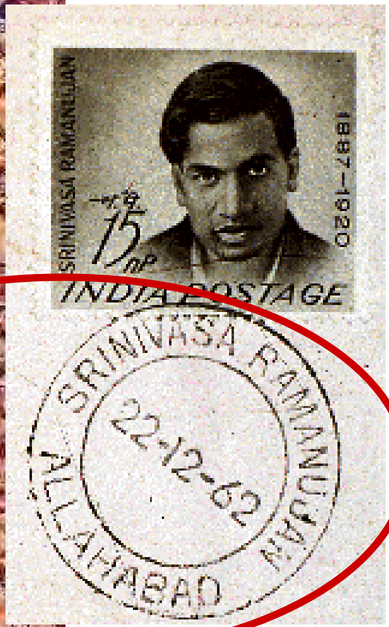
“This "quasi-experimental" approach to proof can help to de-emphasis a focus on rigor and formality for its own sake, and to instead support the view expressed by Hadamard when he stated “The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”

George Polya (1887-1985)

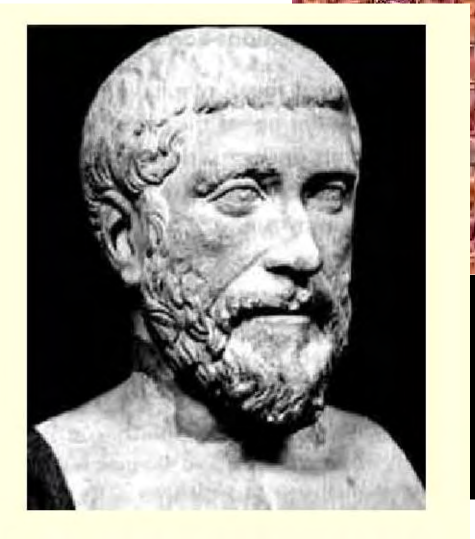
George Polya in ***Mathematical discovery***: *On understanding, learning, and teaching problem solving* (Combined Ed.), New York, Wiley, 1981.

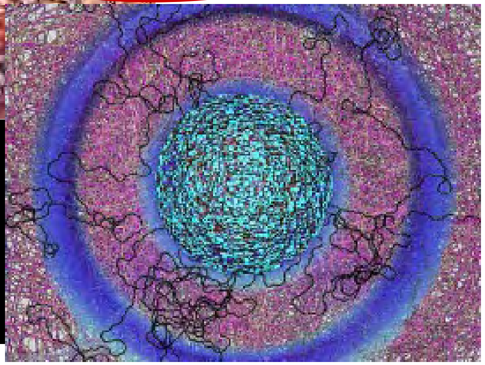


Circles

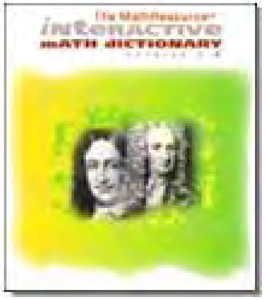


1887/87
1920/85

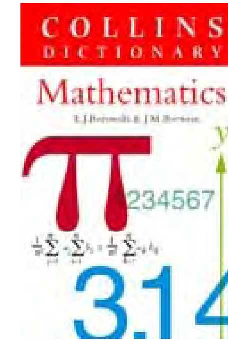




Pi is not in this lecture (much) but it could be



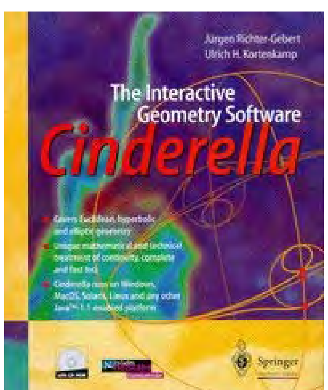
ABSTRACT



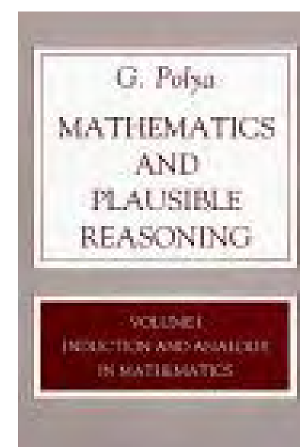
"RESOURCES not COURSES"

Current and expected advances in mathematical computation and scientific visualization make it now possible to do mathematics in many varied and flexible ways.

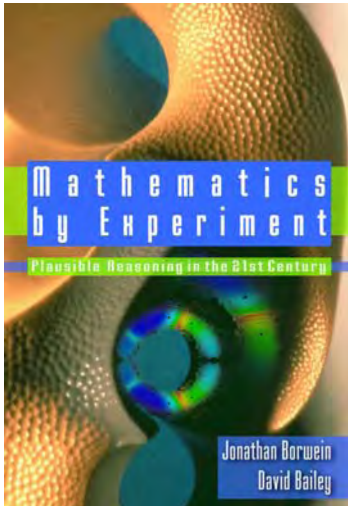
In this talk, I'll explore the opportunities to integrate computational, graphic and other tools into our work --- for philosophic, pedagogic and aesthetic reasons.



URLS http://projects.cs.dal.ca/ddrive
http://users.cs.dal.ca/~jborwein
http://www.experimentalmath.info
http://www.mathresources.com



I. Experimental Mathematics a Philosophical Introduction



FOUR FORMS of EXPERIMENTS

Kantian examples: generating "*the classical non-Euclidean geometries (hyperbolic, elliptic) by replacing Euclid's axiom of parallels (or something equivalent to it) with alternative forms.*"

The Baconian experiment is a contrived as opposed to a natural happening, it *"is the consequence of `trying things out' or even of merely messing about."*

Aristotelian demonstrations: *"apply electrodes to a frog's sciatic nerve, and lo, the leg kicks; always precede the presentation of the dog's dinner with the ringing of a bell, and lo, the bell alone will soon make the dog dribble."*

The most important is **Galilean**: "a *critical experiment -- one that discriminates between possibilities and, in doing so, either gives us confidence in the view we are taking or makes us think it in need of correction."*

the only form which will make Experimental Mathematics a serious enterprise.

From Peter Medawar (1915-87) *Advice to a Young Scientist* (1979)

A Paraphrase of Hersh's Humanism

1. **Mathematics is human**. It is part of and fits into human culture. It does not match Frege's concept of an abstract, timeless, tenseless, objective reality.
2. **Mathematical knowledge is Fallible.** As in science, mathematics can advance by making mistakes and then correcting or even re-correcting them. The "fallibilism" of mathematics is brilliantly argued in Lakatos' *Proofs and Refutations.*
3. **There are different versions of proof or rigo**r. Standards of rigor can vary depending on time, place, and other things. The use of computers in formal proofs, exemplified by the computer-assisted proof of the four color theorem in 1977 (1997), is just one example of an emerging nontraditional standard of rigor.
4. **Empirical evidence, numerical experimentation and probabilistic proof all can help us decide what to believe in mathematics**. Aristotelian logic isn't necessarily always the best way of deciding.
5. **Mathematical objects are a special variety of a social-cultural-historical object**. Contrary to the assertions of certain post-modern detractors, mathematics cannot be dismissed as merely a new form of literature or religion. Nevertheless, many mathematical objects can be seen as shared ideas, like Moby Dick in literature, or the Immaculate Conception in religion.

``Fresh Breezes in the Philosophy of Mathematics'', *MAA Monthly,* Aug 1995, 589-594.

A Paraphrase of Ernest's Social Constructivism

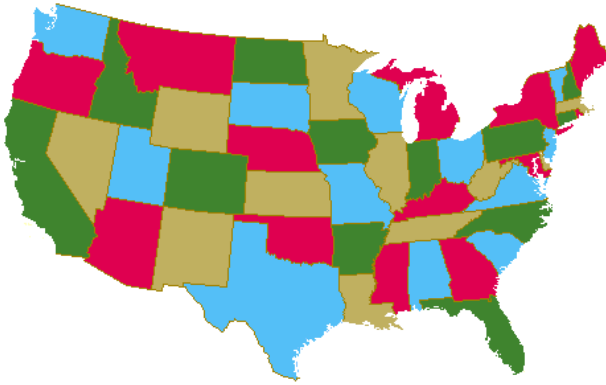
The idea that what is accepted as mathematical knowledge is, to some degree, dependent upon a community's methods of knowledge acceptance is central to the social constructivist school of mathematical philosophy.

The social constructivist thesis is that mathematics is a social construction, a cultural product, fallible like any other branch of knowledge (Paul Ernest)

Associated most notably with his *Social Constructivism as a Philosophy of Mathematics,* Ernest, an English Math-ematician and Professor in Philosophy of Mathematics Education, carefully traces the intellectual pedigree for his thesis, a pedigree that encompasses the writings of Witt-genstein, Lakatos, Davis, and Hersh among others, social constructivism seeks to define mathematical knowledge and epistemology through social structure and interaction of the mathematical community and society as a whole.

DISCLAIMER: Social Constructivism is not Cultural Relativism

Grand Challenges in Mathematics (CISE 2000)

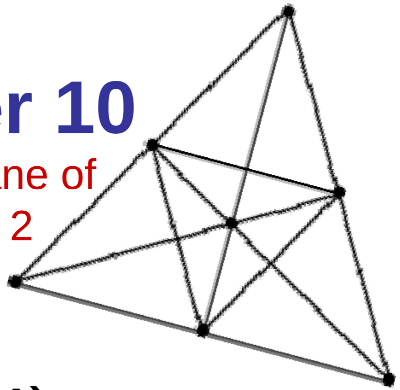


Are few and far between

- **Four Colour Theorem** (1976,1997)
- **Kepler's problem** (Hales, 2004-11)

On next slide

- Nonexistence of **Projective Plane of Order 10**
  - **10^2+10+1** lines and points on each other (n+1 fold)
    - 2000 Cray hrs in 1990
    - next similar case:18 needs10^{12} hours?



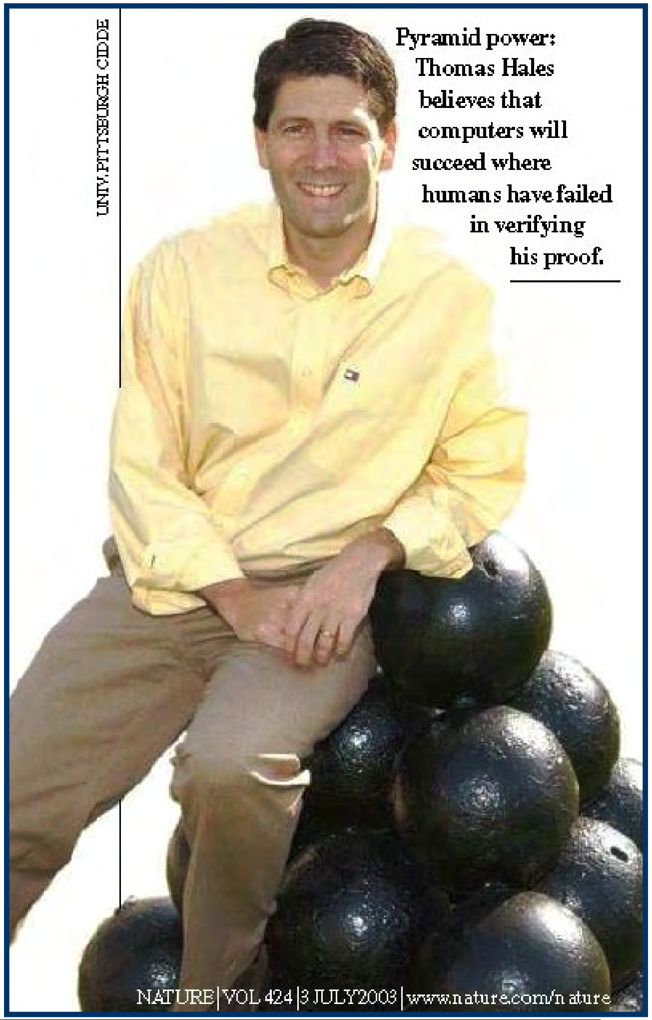
Fano plane of order 2

- **Fermat's Last Theorem** (Wiles 1993, 1994)
  - By contrast, any counterexample was too big to find (1985)

$$x^N + y^N = z^N, N > 2$$

has only trivial integer solutions

- **Kepler's** conjecture the densest way to stack spheres is in a pyramid
  - oldest problem in discrete geometry?
  - most interesting recent example of computer assisted proof
  - published in *Annals of Mathematics* with an "only 99% checked" disclaimer
  - Many varied reactions. *In Math, Computers Don't Lie. Or Do They?* (NYT, 6/4/04)



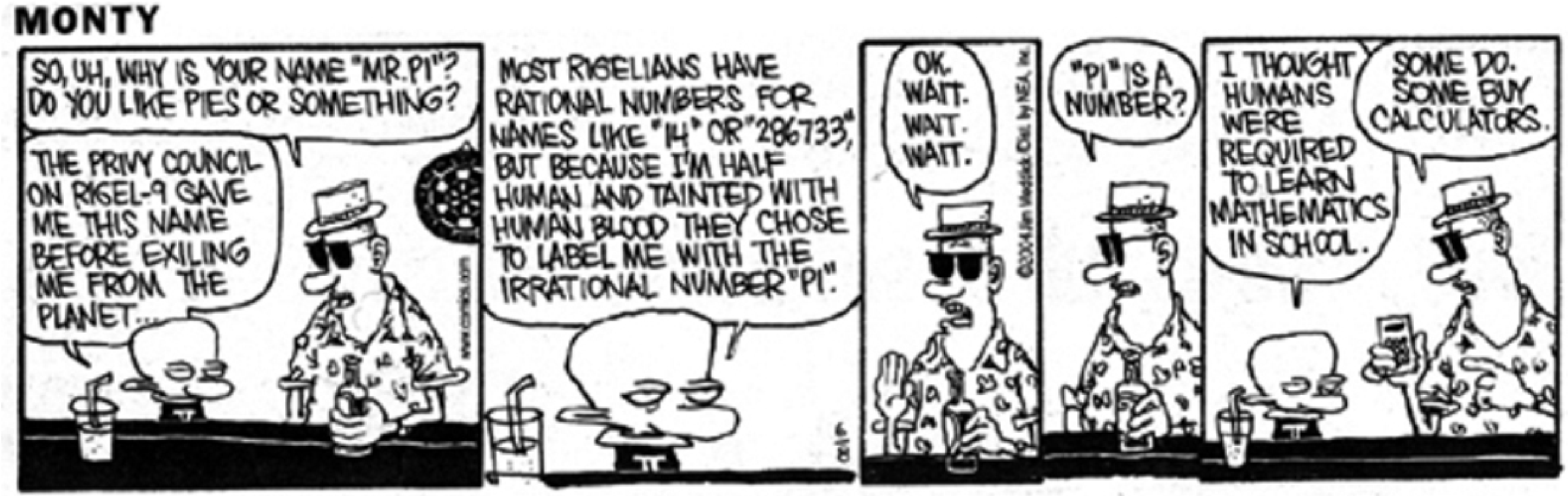
- **Famous earlier examples**: Four Color Theorem and Non-existence of a Projective Plane of Order 10.
  - the three raise quite distinct questions - both real and specious
  - as does status of classification of Finite Simple Groups

Formal Proof theory (code validation) has received an unexpected boost: automated proofs *may* now exist of the Four Color Theorem and Prime Number Theorem

• COQ: *When is a proof a proof* ? Economist, April 2005

Cultural Mathematics

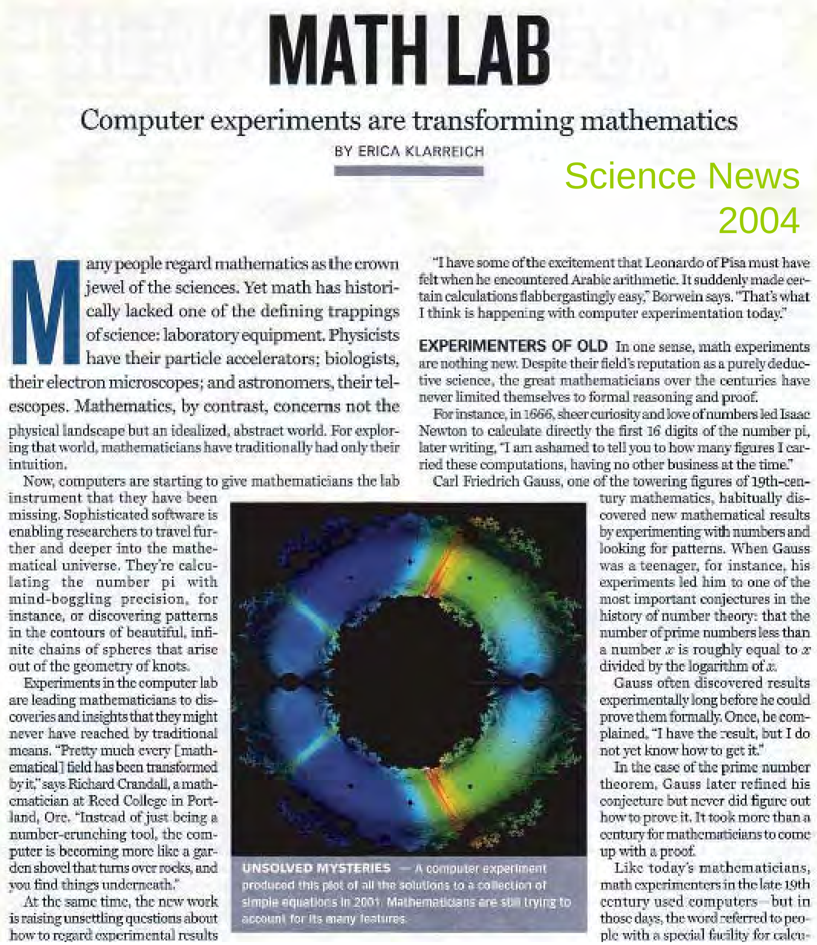
Mr Pi


MONTY
SO, UH, WHY IS YOUR NAME "MR. PI"? DO YOU LIKE PIES OR SOMETHING?
THE PRIVY COUNCIL ON RIGEL-9 GAVE ME THIS NAME BEFORE EXILING ME FROM THE PLANET...
MOST RIGELIANS HAVE RATIONAL NUMBERS FOR NAMES LIKE "14" OR "286733", BUT BECAUSE I'M HALF HUMAN AND TAINTED WITH HUMAN BLOOD THEY CHOSE TO LABEL ME WITH THE IRRATIONAL NUMBER "PI".
OK. WAIT. WAIT. WAIT.
"PI" IS A NUMBER?
I THOUGHT HUMANS WERE REQUIRED TO LEARN MATHEMATICS IN SCHOOL.
SOME DO. SOME BUY CALCULATORS.

Experimental Mathodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results

Detailed examples are given in Chapter 1



MATH LAB

Computer experiments are transforming mathematics

BY ERICA KLARREICH

Many people regard mathematics as the crown jewel of the sciences. Yet math has historically lacked one of the defining trappings of science: laboratory equipment. Physicists have their particle accelerators; biologists, their electron microscopes; and astronomers, their telescopes. Mathematics, by contrast, concerns not the physical landscape but an idealized, abstract world. For exploring that world, mathematicians have traditionally had only their intuition.

Now, computers are starting to give mathematicians the lab instrument that they have been missing. Sophisticated software is enabling researchers to travel further and deeper into the mathematical universe. They're calculating the number pi with mind-boggling precision, for instance, or discovering patterns in the contours of beautiful, infinite chains of spheres that arise out of the geometry of knots.

Experiments in the computer lab are leading mathematicians to discoveries and insights that they might never have reached by traditional means. "Pretty much every [mathematical] field has been transformed by it," says Richard Crandall, a mathematician at Reed College in Portland, Ore. "Instead of just being a number-crunching tool, the computer is becoming more like a garden shovel that turns over rocks, and you find things underneath."

At the same time, the new work is raising unsettling questions about how to regard experimental results

UNSOLVED MYSTERIES — A computer experiment produced this plot of all the solutions to a collection of simple equations in 2001. Mathematicians are still trying to account for its many features.

"I have some of the excitement that Leonardo of Pisa must have felt when he encountered Arabic arithmetic. It suddenly made certain calculations flabbergastingly easy," Borwein says. "That's what I think is happening with computer experimentation today."

EXPERIMENTERS OF OLD In one sense, math experiments are nothing new. Despite their field's reputation as a purely deductive science, the great mathematicians over the centuries have never limited themselves to formal reasoning and proof.

For instance, in 1666, sheer curiosity and love of numbers led Isaac Newton to calculate directly the first 16 digits of the number pi, later writing, "I am ashamed to tell you to how many figures I carried these computations, having no other business at the time."

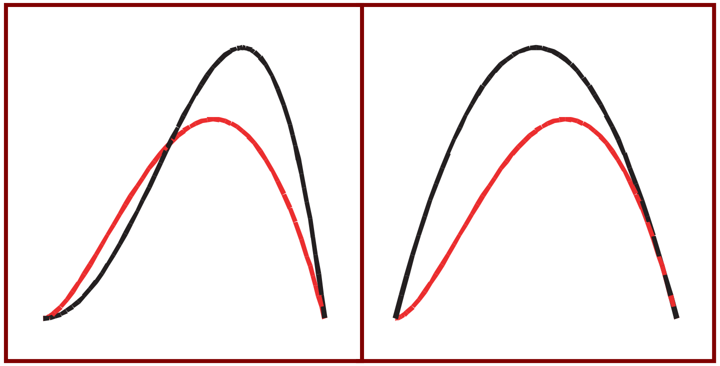
Carl Friedrich Gauss, one of the towering figures of 19th-century mathematics, habitually discovered new mathematical results by experimenting with numbers and looking for patterns. When Gauss was a teenager, for instance, his experiments led him to one of the most important conjectures in the history of number theory: that the number of prime numbers less than a number x is roughly equal to x divided by the logarithm of x.

Gauss often discovered results experimentally long before he could prove them formally. Once, he complained, "I have the result, but I do not yet know how to get it."

In the case of the prime number theorem, Gauss later refined his conjecture but never did figure out how to prove it. It took more than a century for mathematicians to come up with a proof.

Like today's mathematicians, math experimenters in the late 19th century used computers—but in those days, the word referred to people with a special facility for calcu-

Science News 2004



Comparing $-y^2\ln(y)$ (red) to $y-y^2$ and y^2-y^4

"It says it's sick of doing things like inventories and payrolls, and it wants to make some break-throughs in astrophysics."

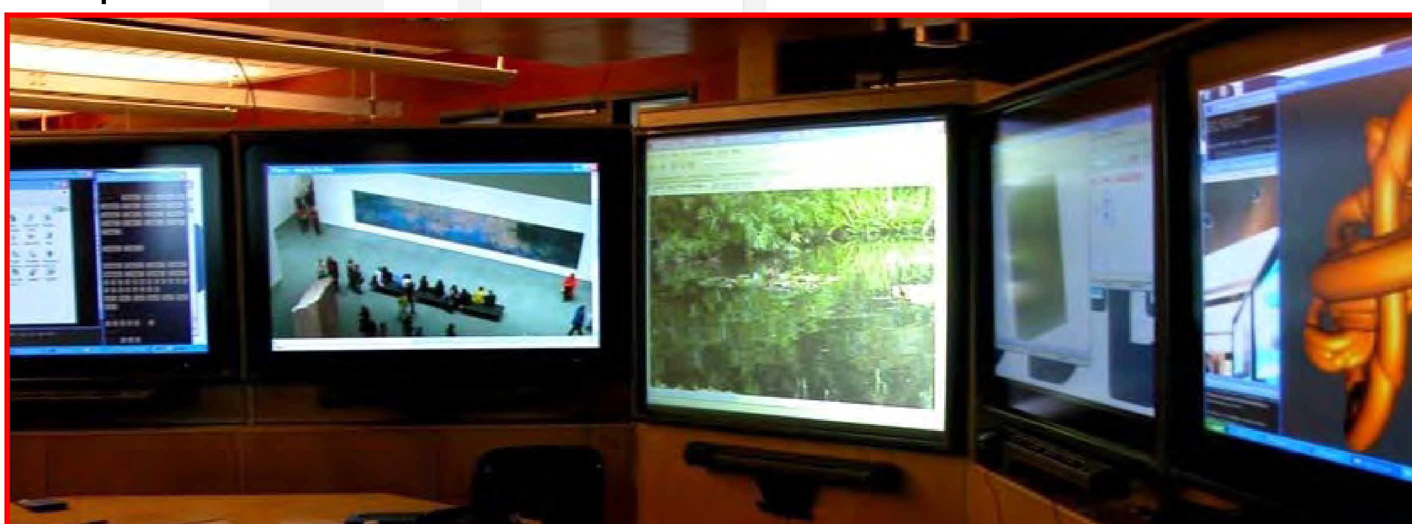
Collaboration in 2006

Collaboration goes National: East meets West

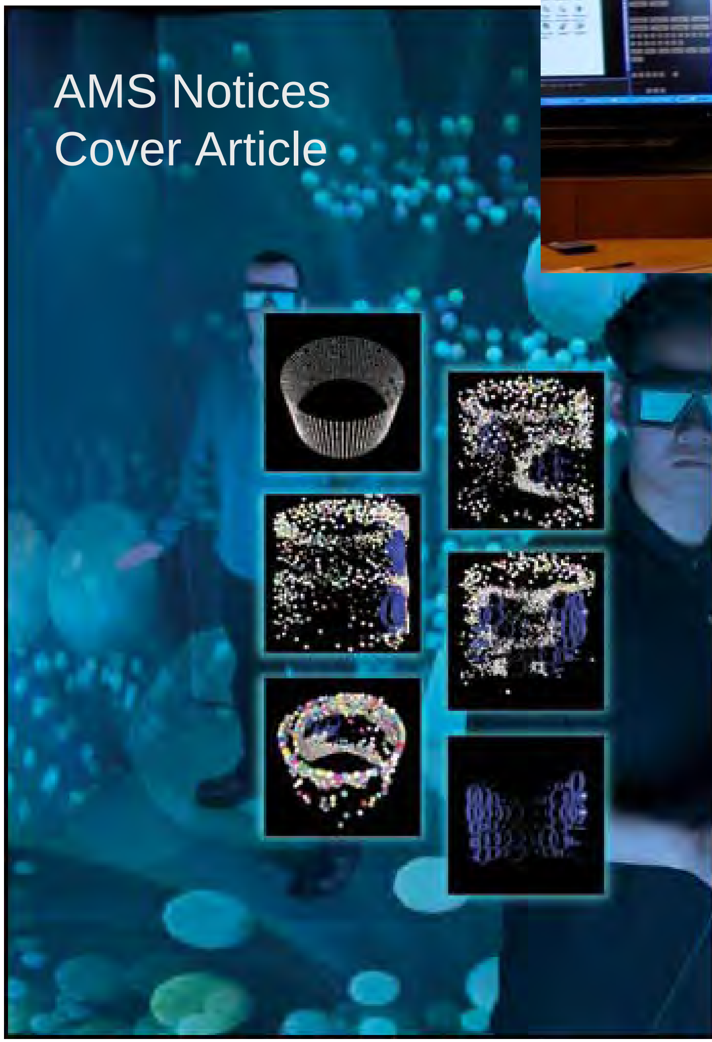
Welcome to D-DRIVE whose mandate is to study and develop resources specific to distributed research in the sciences with first client groups being the following communities

- High Performance Computing
- Mathematical and Computational Science Research
- Science Outreach
  - ✓ Educational
  - ✓ Research

To the desktop and lecture theatre



5 SMART Touch-sensitive Interwoven Screens



Experimental Inductive Mathematics

Our web site:

www.experimentalmath.info

contains all links and references

“Elsewhere Kronecker said ``In mathematics, I recognize true scientific value only in concrete mathematical truths, or to put it more pointedly, only in mathematical formulas.'' ... I would rather say ``computations'' than ``formulas'', but my view is essentially the same.”

Harold Edwards, *Essays in Constructive Mathematics*, 2004

Trans-Canada 'Sea to Sea' Seminar
Tuesdays PST 11.30 MST 12.30 AST 3.30 and even 7.30 GMT
[Sept 28 - PBB on RH]

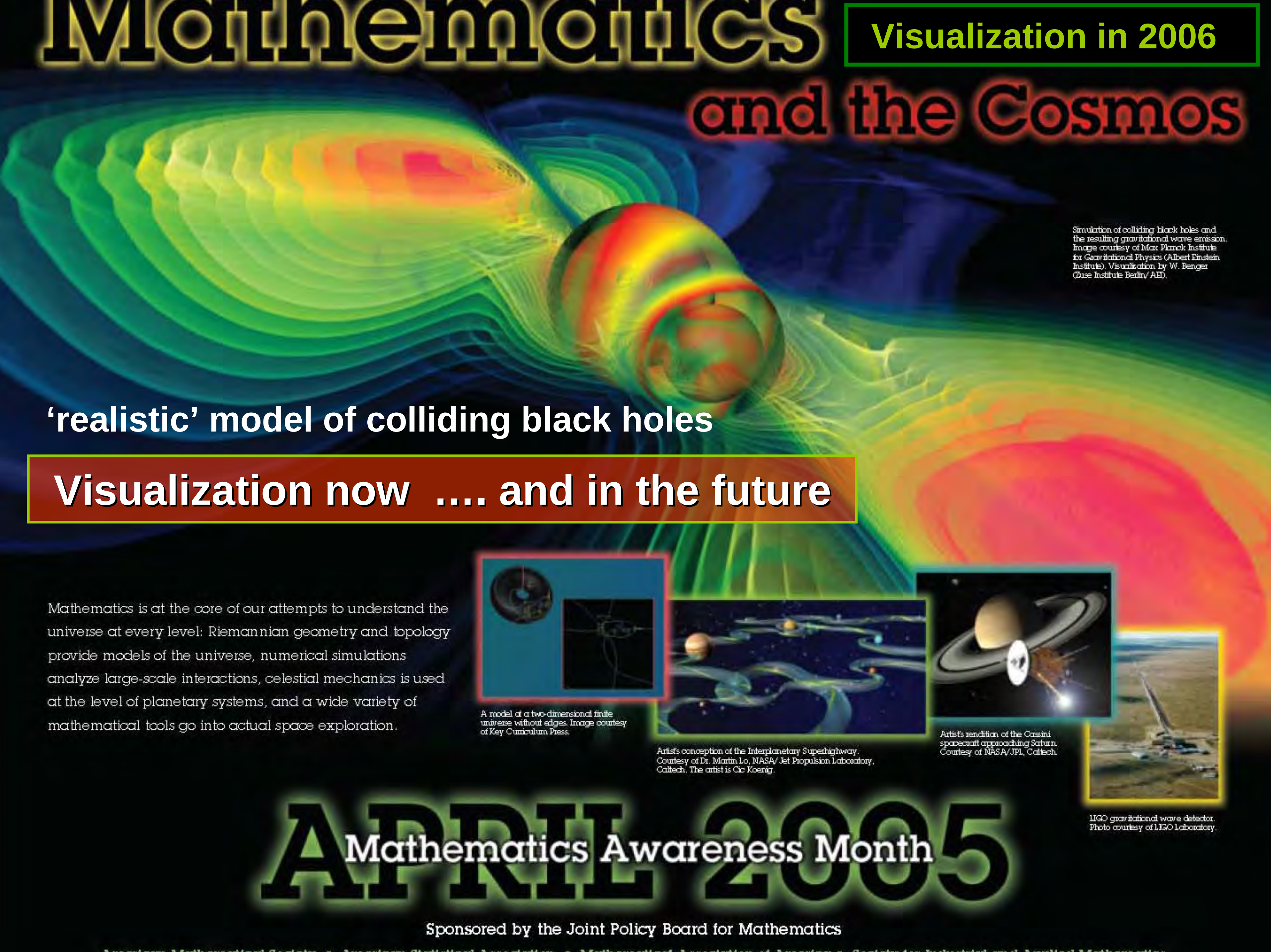
The 2,500 sq-metre IRMACS research centre

The building is a also a 190cpu G5 Grid

At the official April opening, I gave one of the four presentations from D-DRIVE

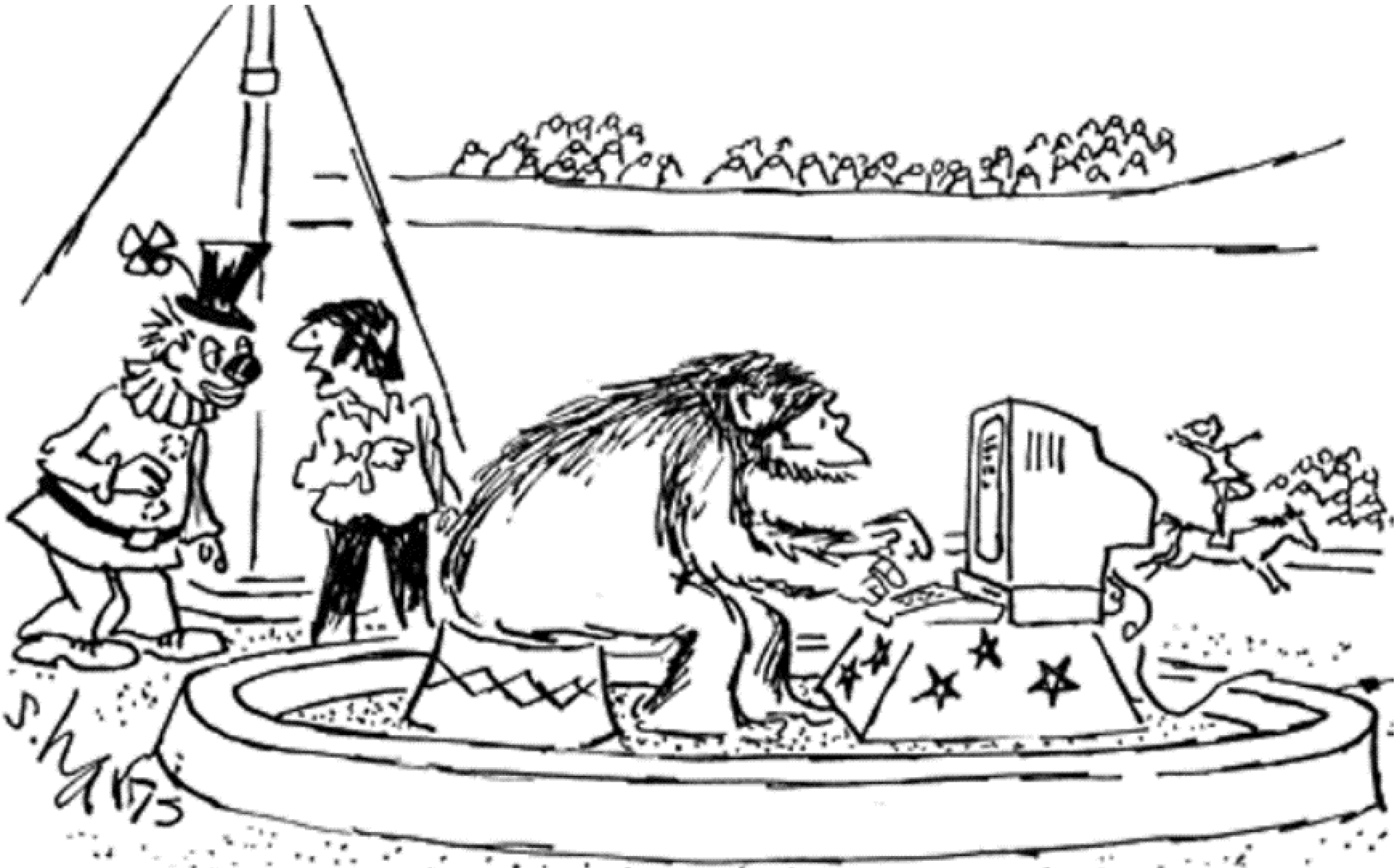

The present
SFU - Colab
Pierre Boulanger
SFU - Colab
University of Alberta
westgrid...
UBC - MAGIC
Netera AG
The AG in Action in CoLab

Visualization in 2006



'realistic' model of colliding black holes

Visualization now …. and in the future

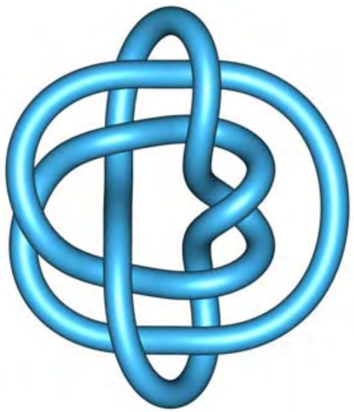


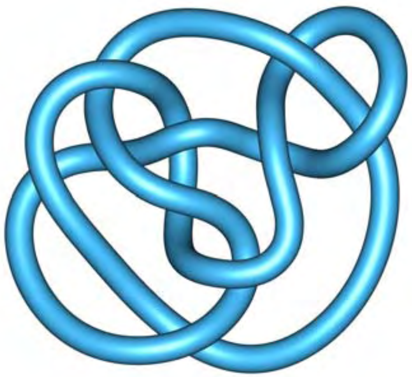
"On the other hand, he never learned to ride a unicycle."

The Perko Pair 10_{161} and 10_{162}

are two adjacent 10-crossing knots (1900)

We need to learn to judge such proofs

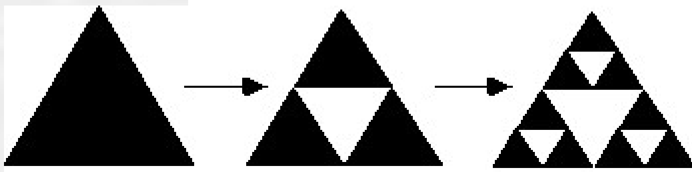
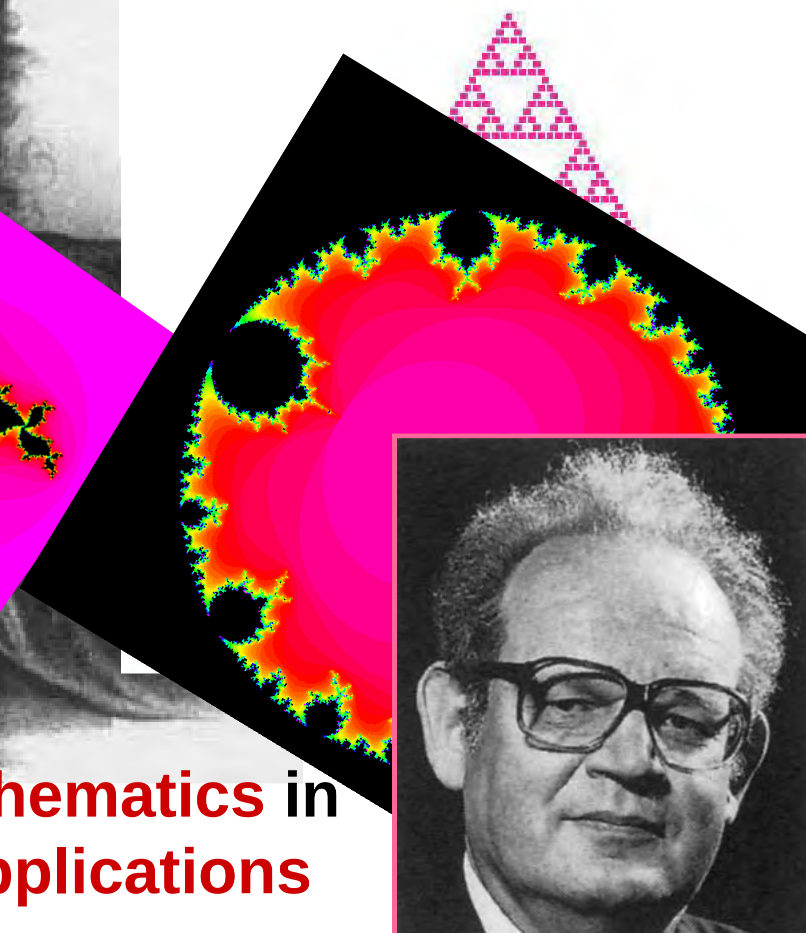
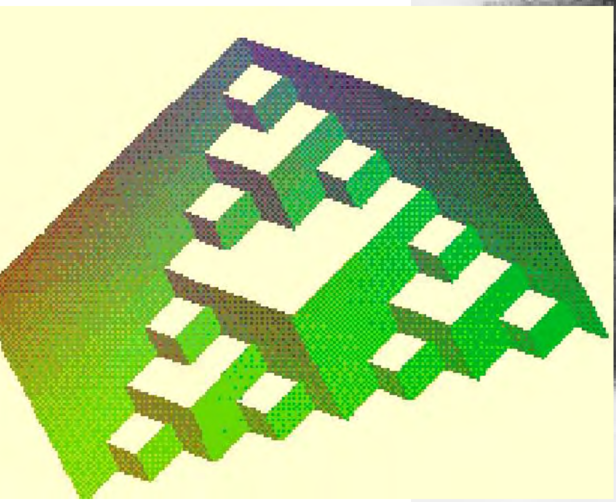




- **first shown to be the same by Ken Perko in 1974**
- **and beautifully made dynamic in KnotPlot (open source)**

Self-Similarity Everywhere

From Pascal
and Sierpinski
to Julia, Fatou
& Mandelbrot

‘cut and fold’

Truly modern mathematics in nature, art and applications

Pascal's Triangle Interface

INSTRUCTIONS **www.cecm.sfu.ca/interfaces**

Output Image

Rows (max 100): 30

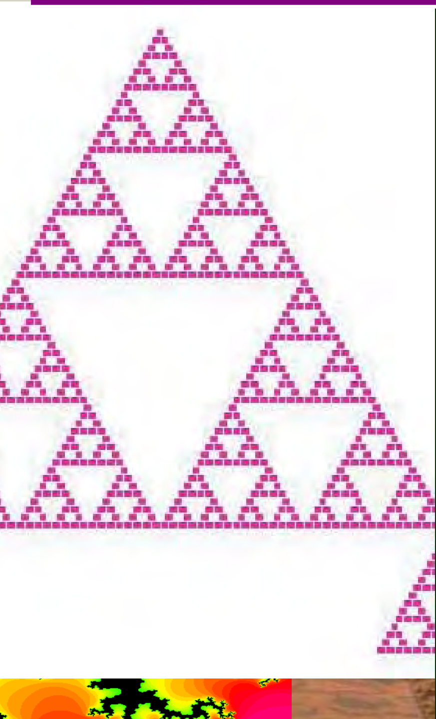
Modulus (2 to 16): 5

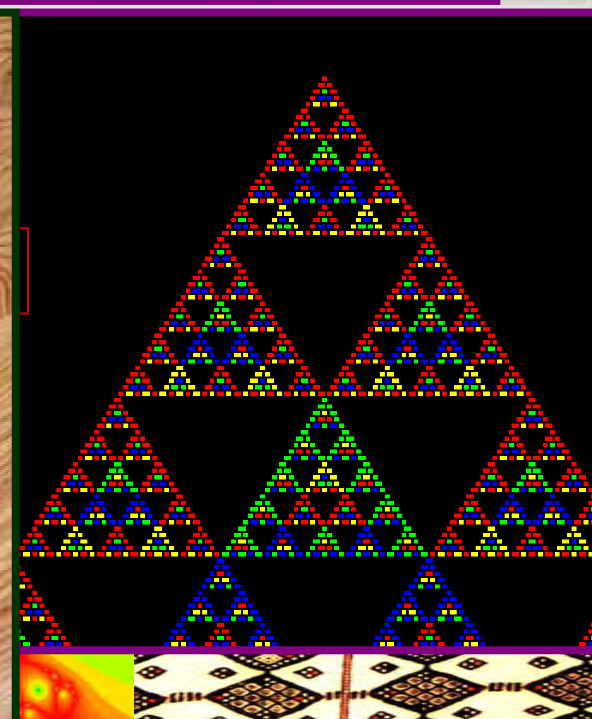
Image size: 300

Deterministic and Random

1   1 1   1 2 1   1 3 3 1   1 4 6 4 1   1 5 10 10 5 1

1 6 15 20 15 6 1   1 7 21 35 21 7 1



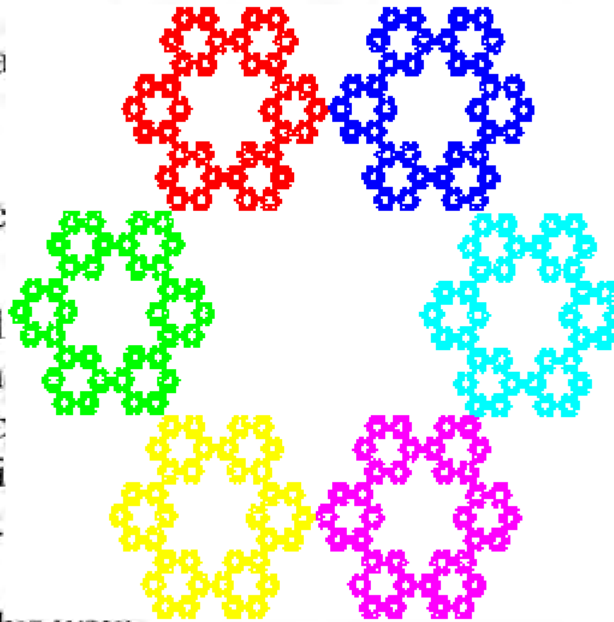


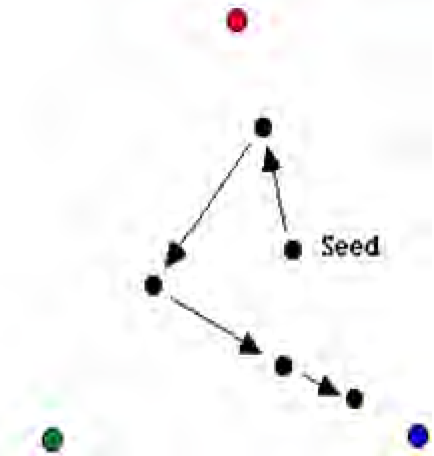
FRACTALINA

About Fractalina

Fractalina allows the input and itera ms. play "the chaos game".

To see it in action, you can go direc



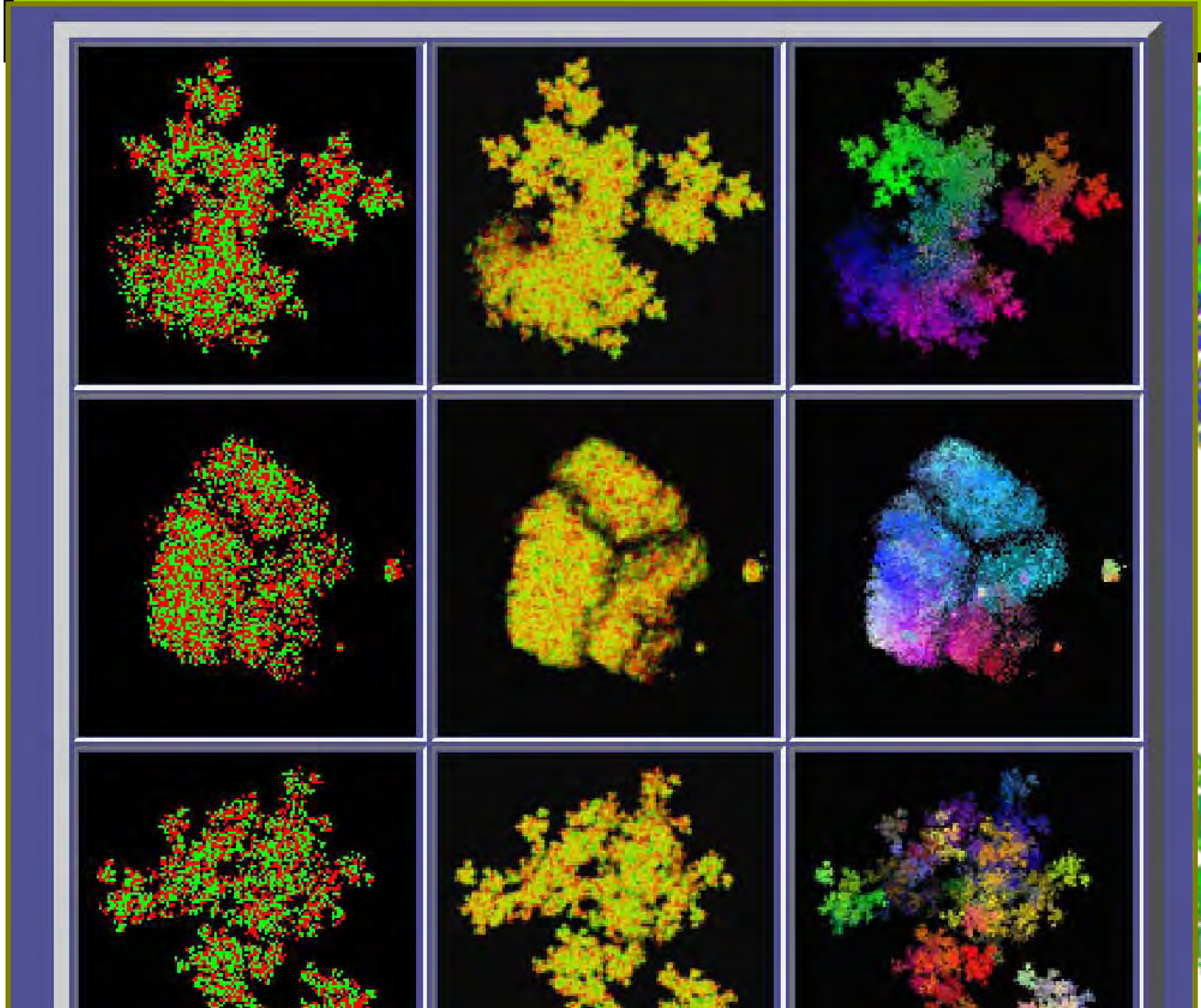


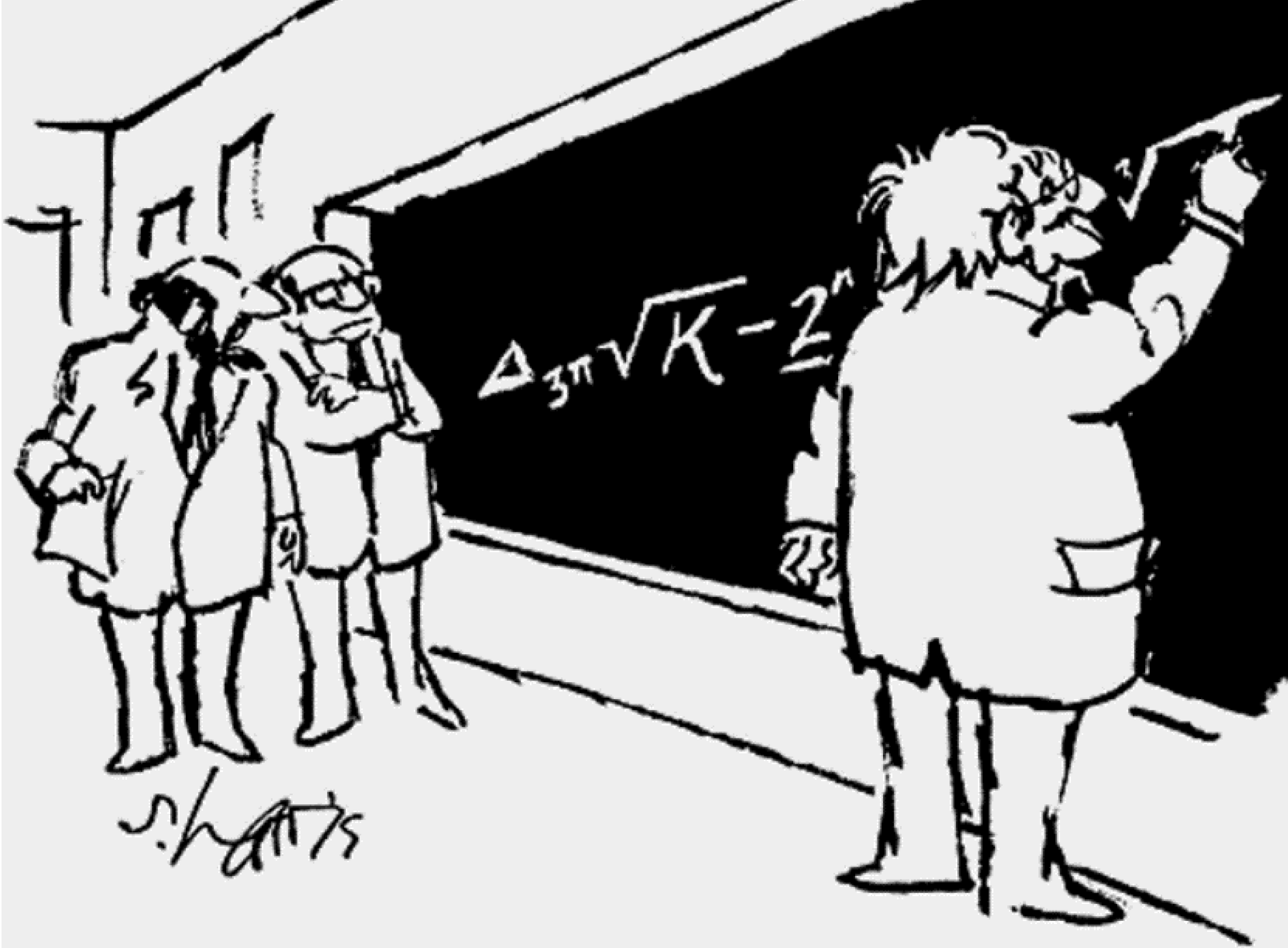
The chaos game begins with the sel e transformations are of a special kind. Each transformation h rotate or compress. We sometimes informally think of the p n lives, so we can say things like "go halfway to the transformati , compression, and position values that determine how it works.

The chaos game can be explained this way:

1. Starting at any point, randomly choose one of the transformations.
2. Go part of the way towards the center point of that transformation and rotate part way around it.
3. Repeat the process from the resulting point.

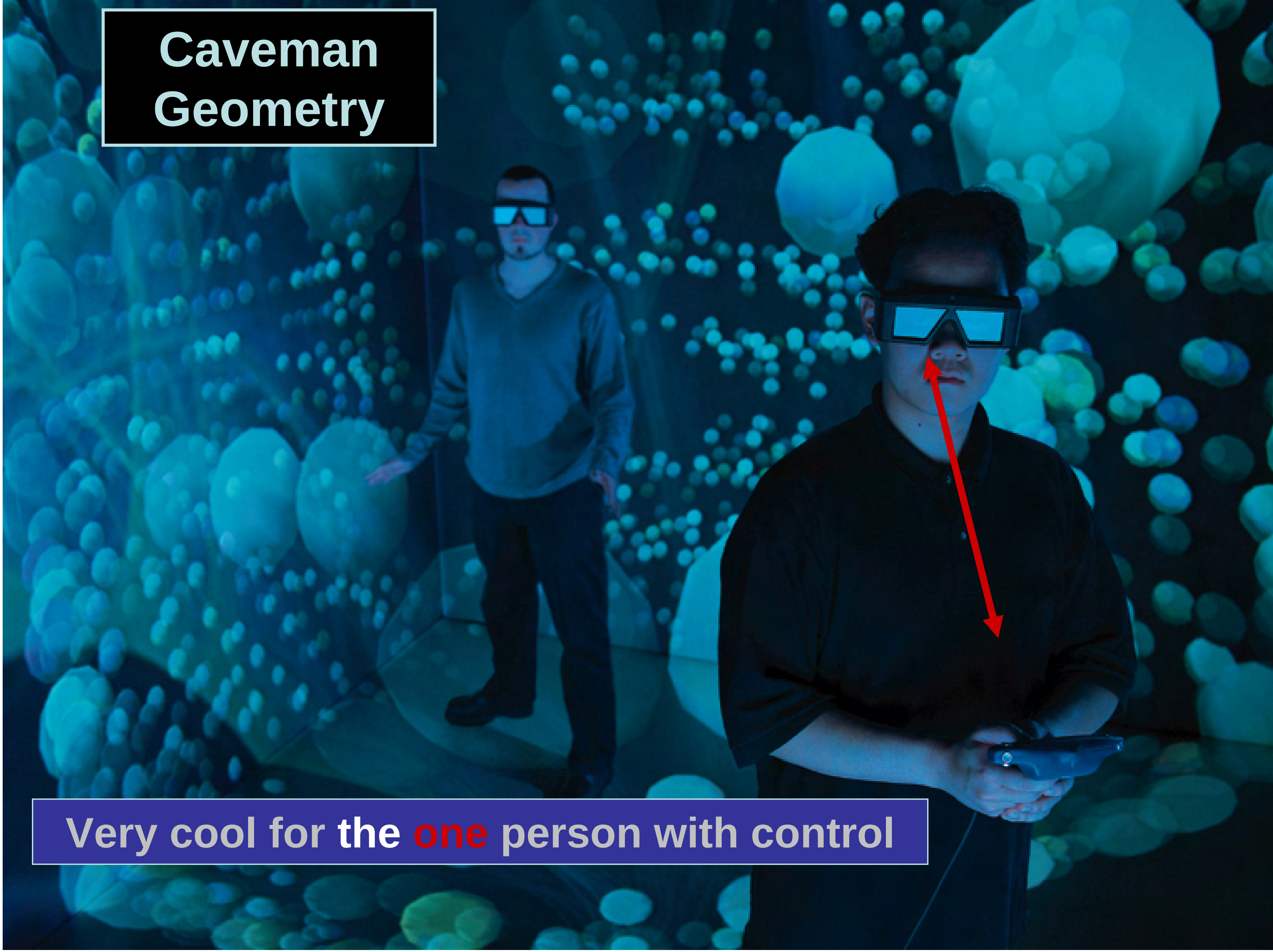
Chaos Games in Genetics





"We have reason to believe Bingleman is an irrational number himself."

Caveman Geometry



Very cool for **the** one person with control

CoLab at SFU

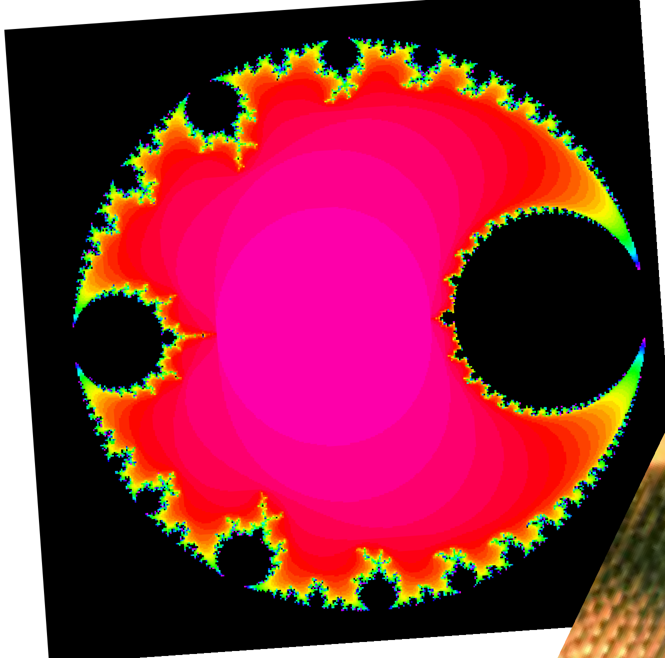
Visualization


All features are active

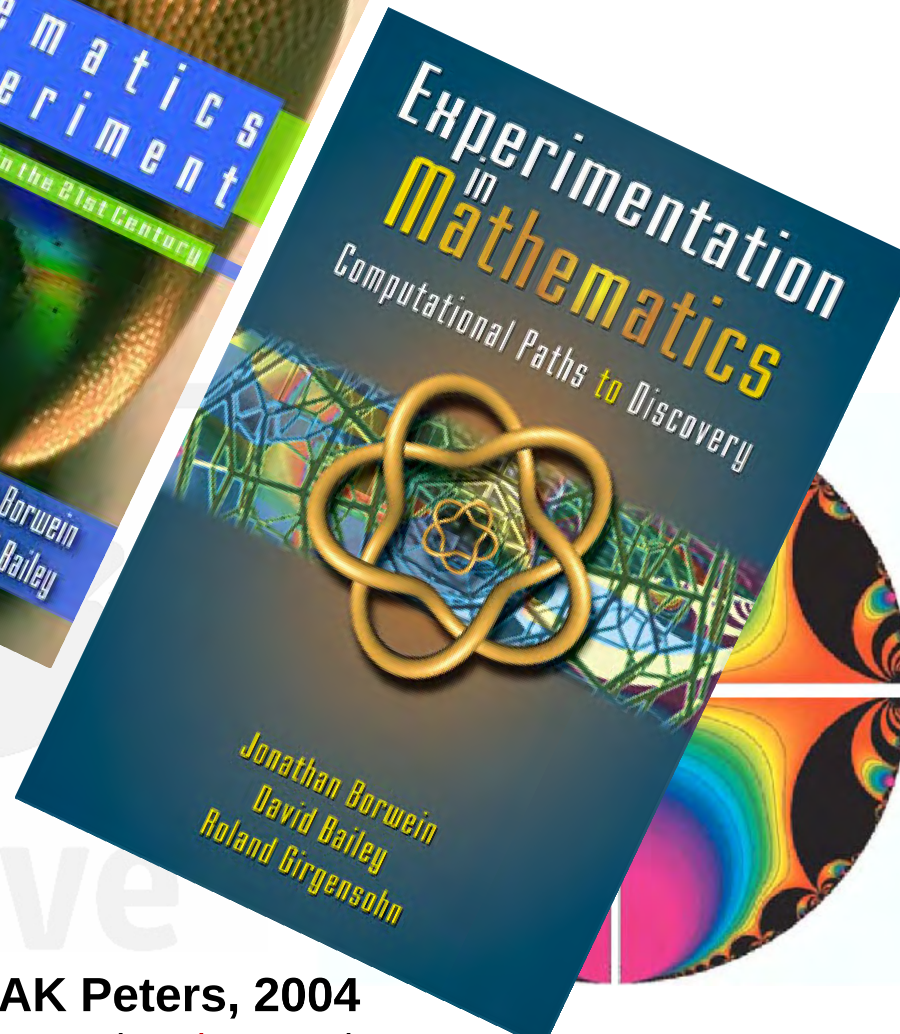
Me and my Avatar (2003)


INSTITUTE of
SOLID GEOMETRY
Institute of
Plane
Geometry

Mathematical Data Mining



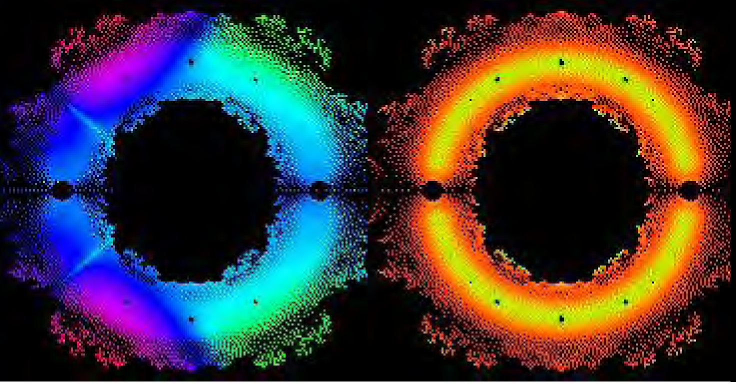
An unusual Mandelbrot parameterization



Three visual examples follow

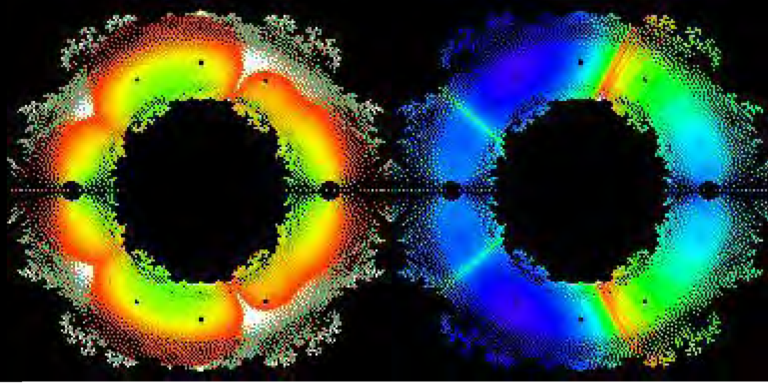
- ✓ Roots of `1/-1’ polynomials
- ✓ Ramanujan’s fractal
- ✓ Indra’s Pearls

AK Peters, 2004
(CD in press)



Roots of Zeros

What you draw is what you see (visible patterns in number theory)



Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

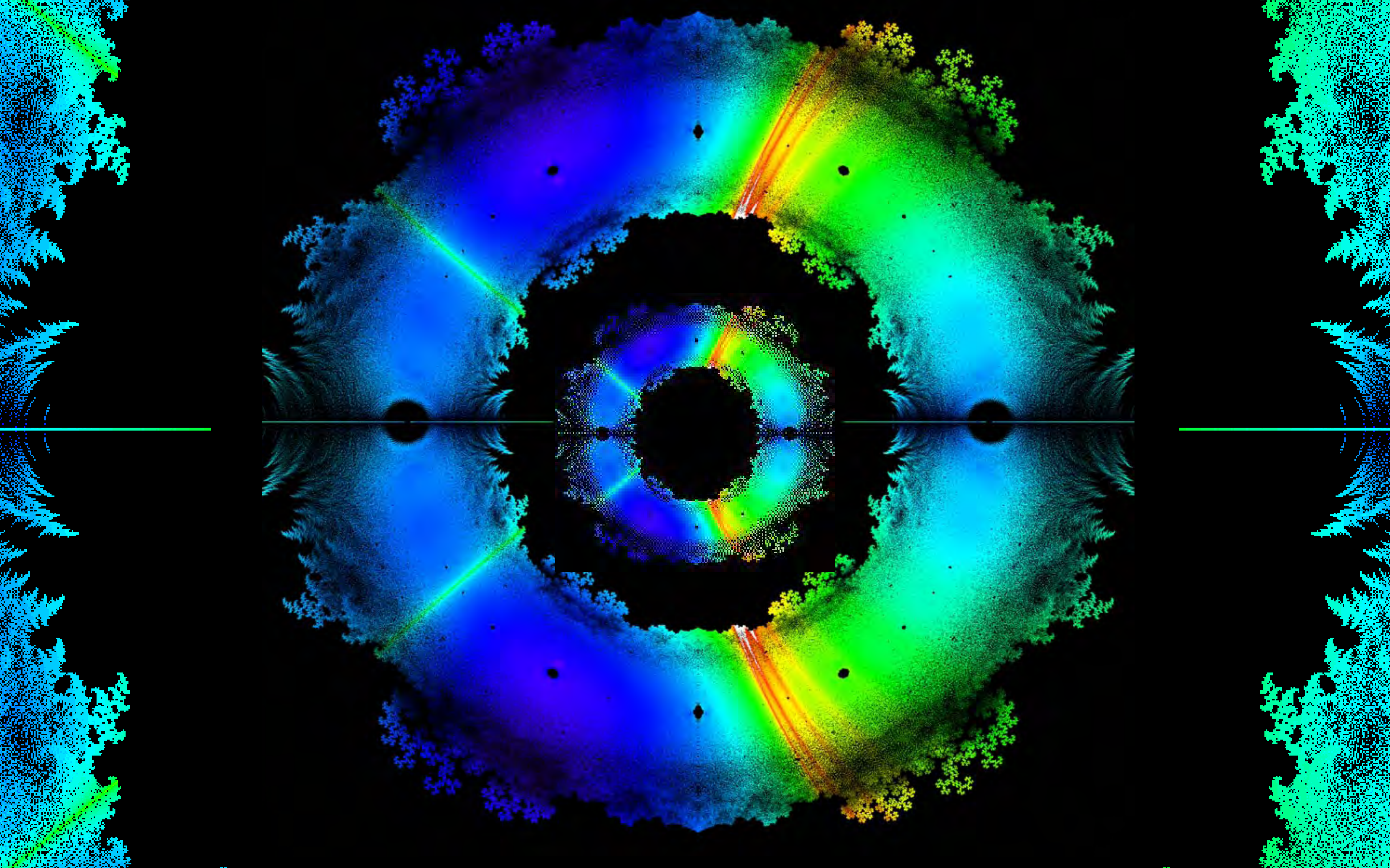
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. The color scale represents a normalized sensitivity to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at **3600 dpi**)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $\mathbf{x}^9$ term
- **The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

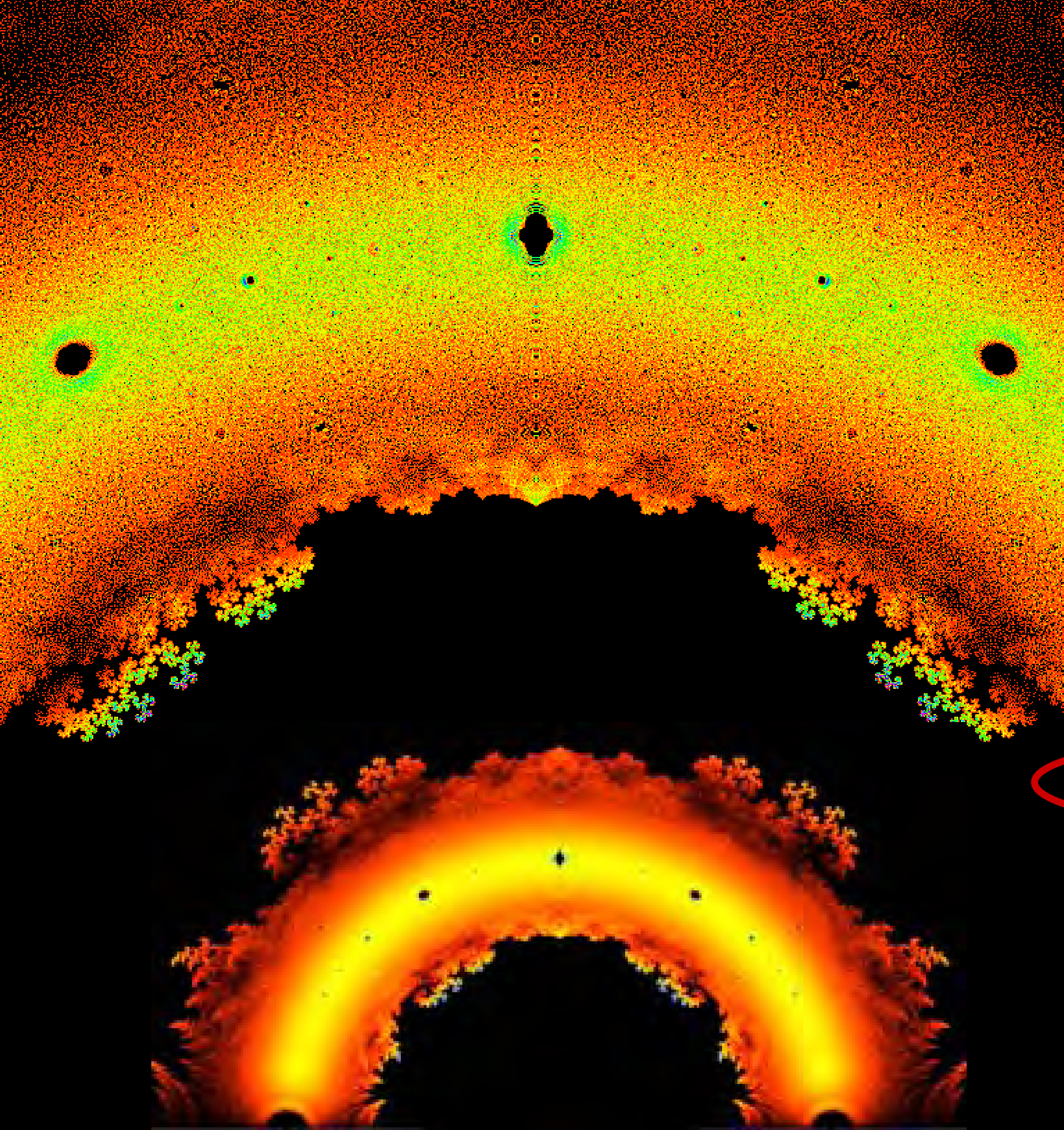
"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!" Greg Chaitin, Interview, 2000.

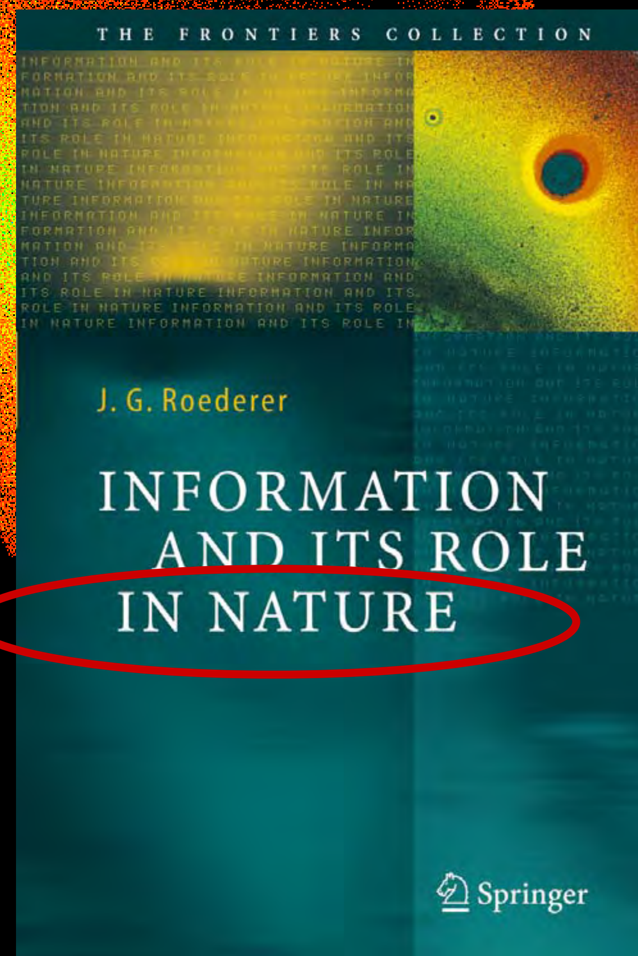
The TIFF on THREE SCALES



Pictures are more democratic but they come from formulae

Roots in the most stable colouring

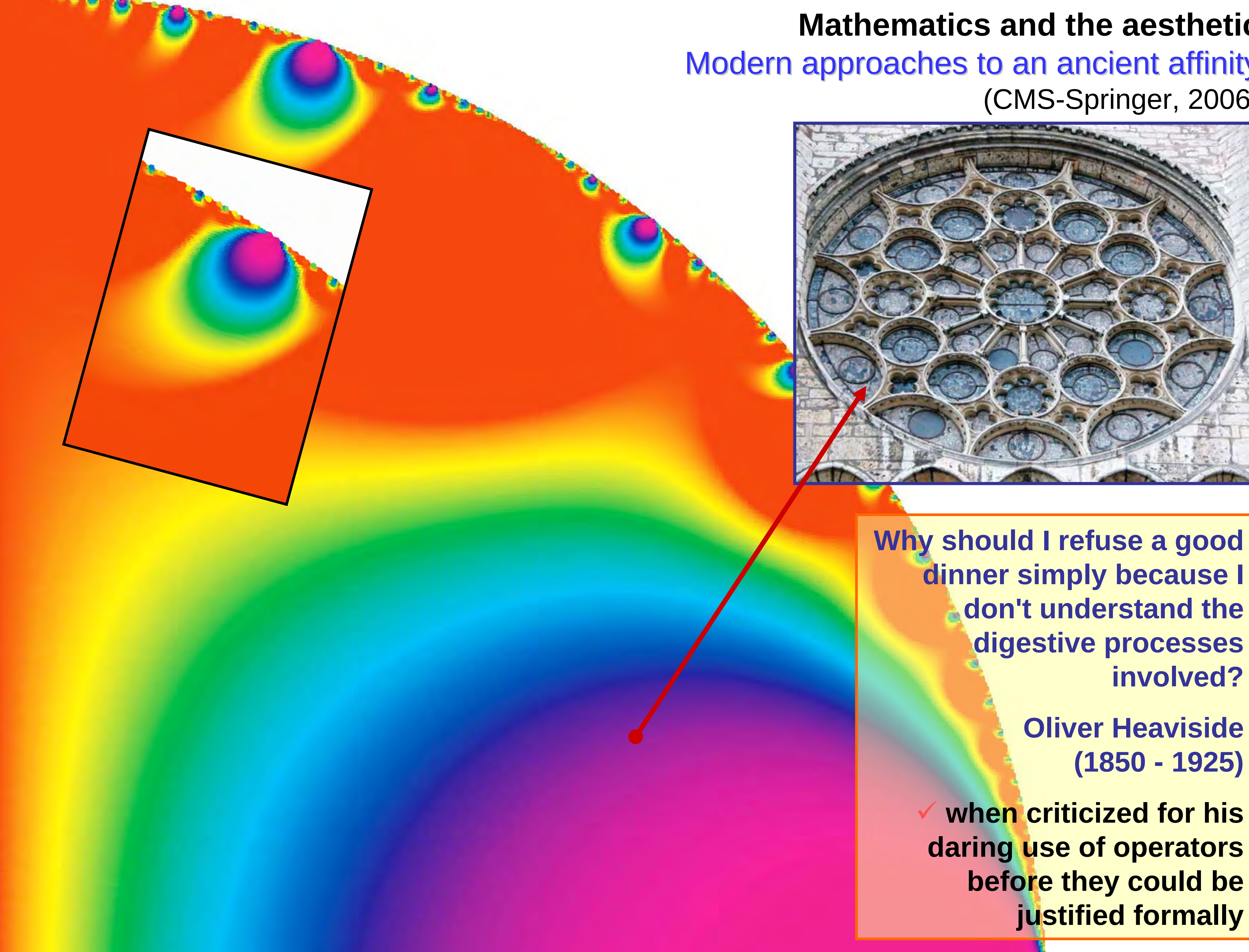




Mathematics and the aesthetic

Modern approaches to an ancient affinity

(CMS-Springer, 2006)

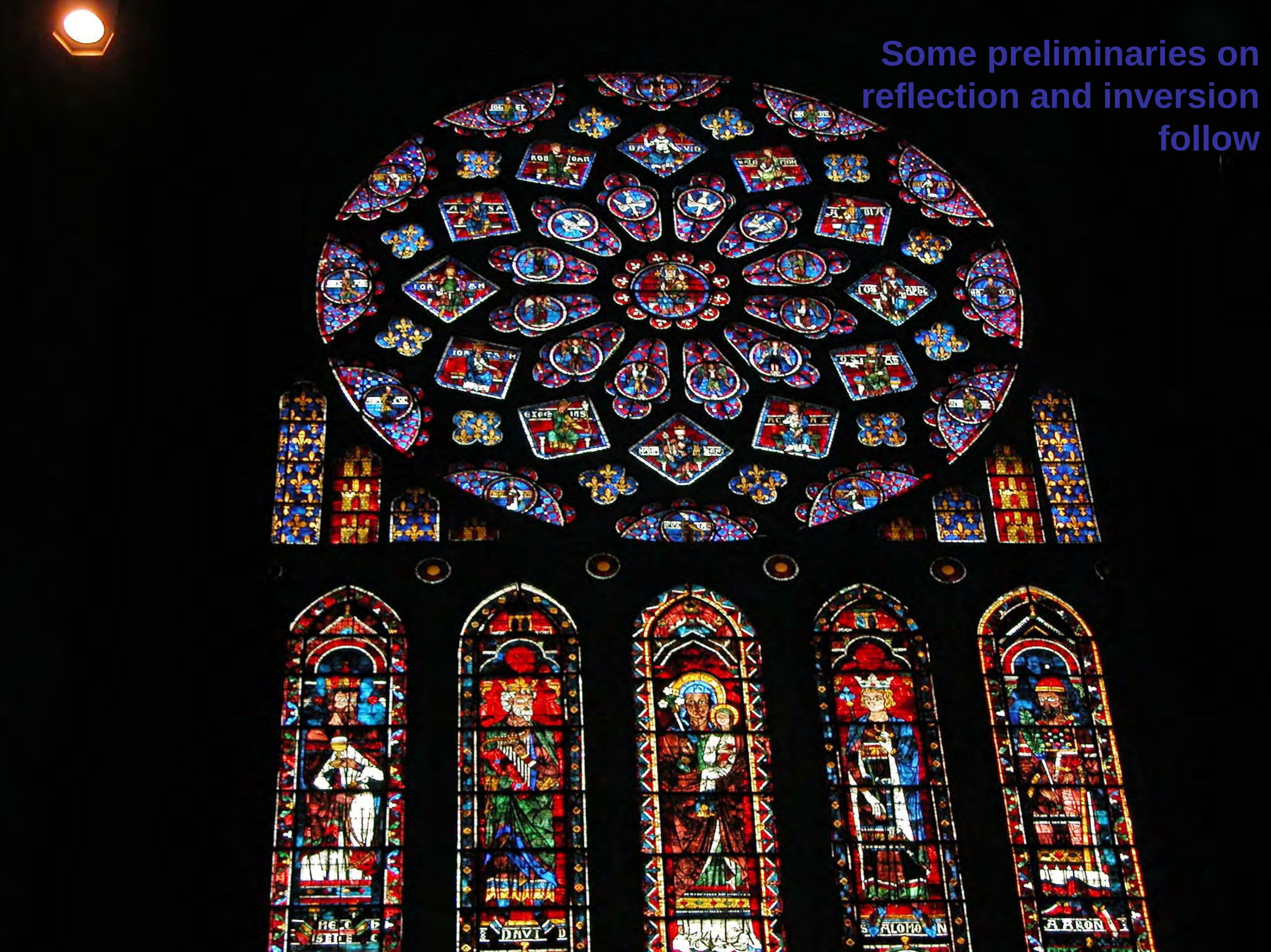


Why should I refuse a good dinner simply because I don't understand the digestive processes involved?

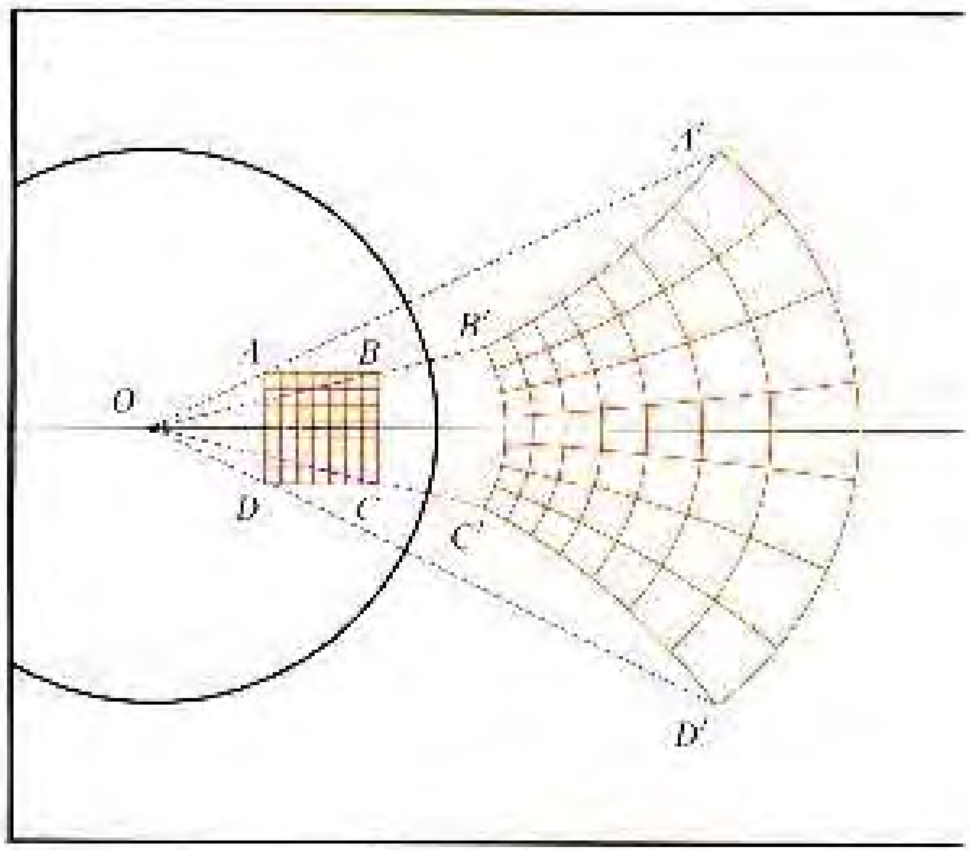
Oliver Heaviside (1850 - 1925)

- **when criticized for his daring use of operators before they could be justified formally**

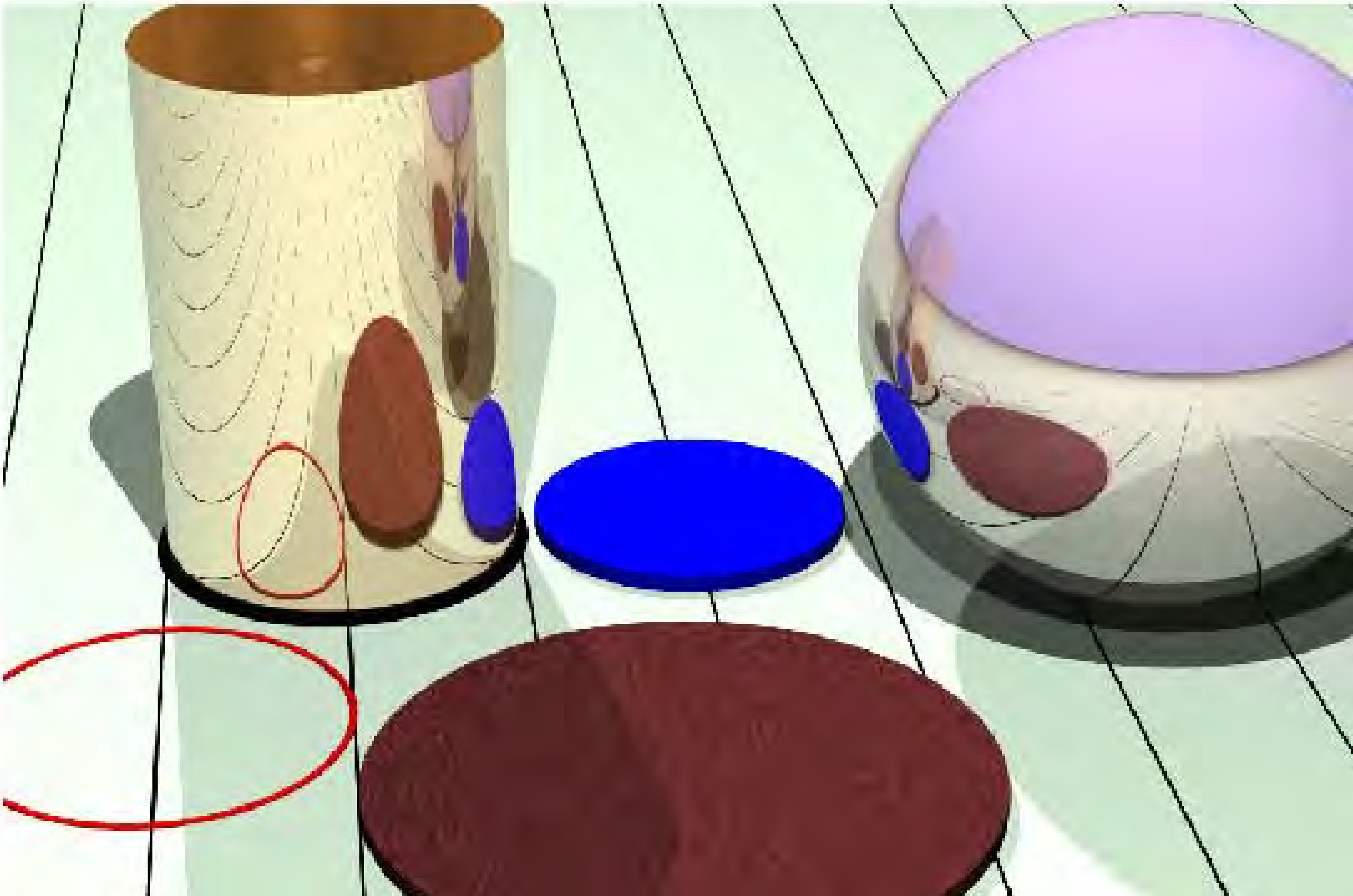
Some preliminaries on reflection and inversion follow



Inversion in a Circle:

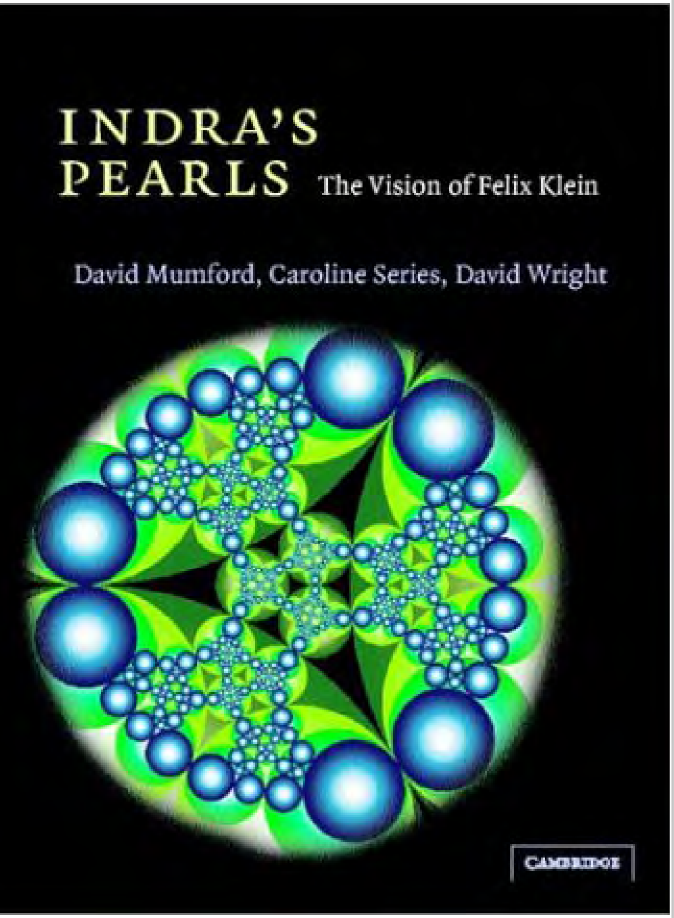


(Euclidean) Reflection in a Circle:

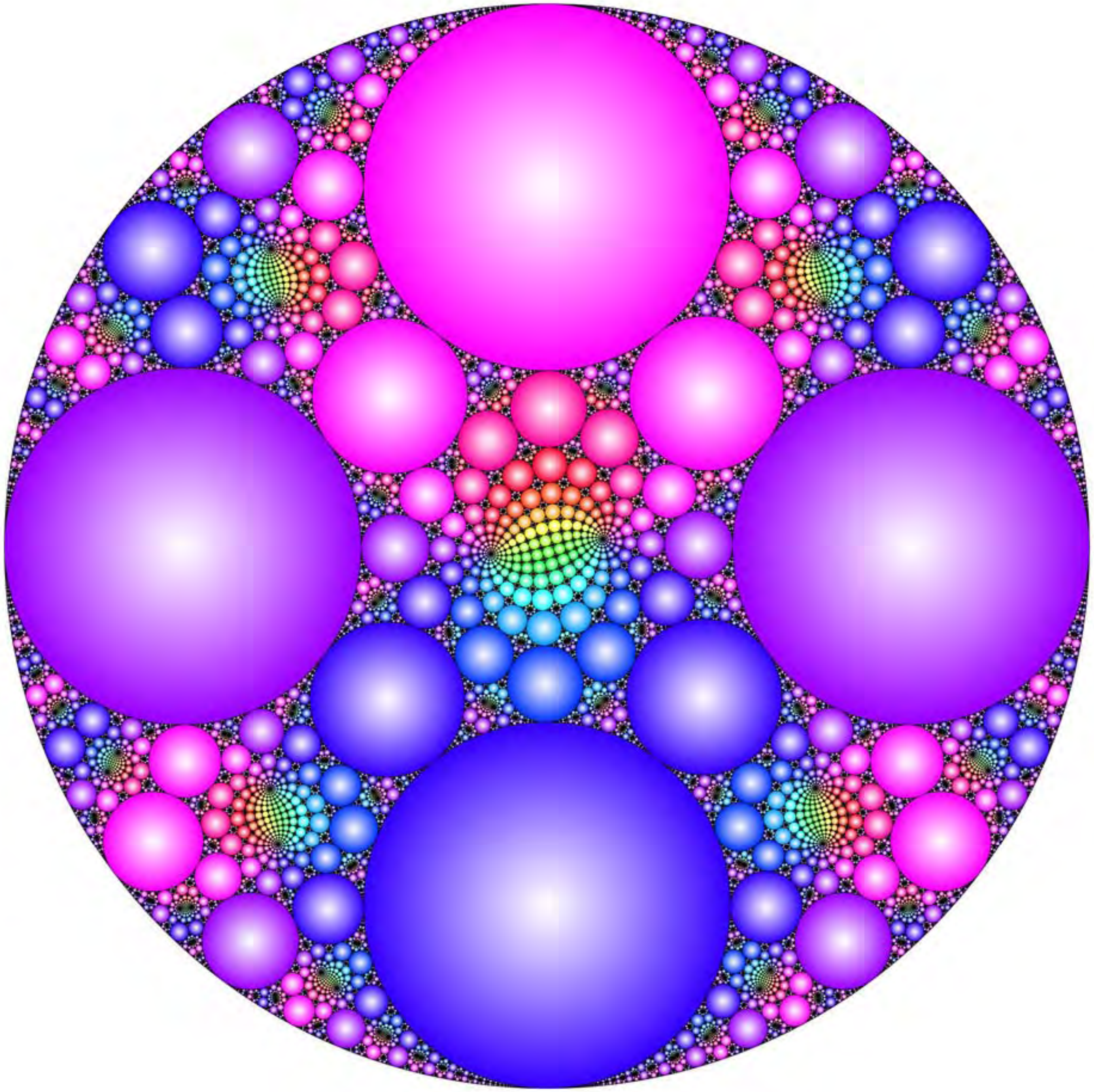


Indra's Pearls

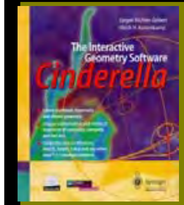
A merging of 19th and 21st Centuries



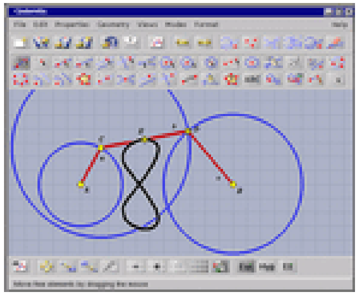
Double cusp group



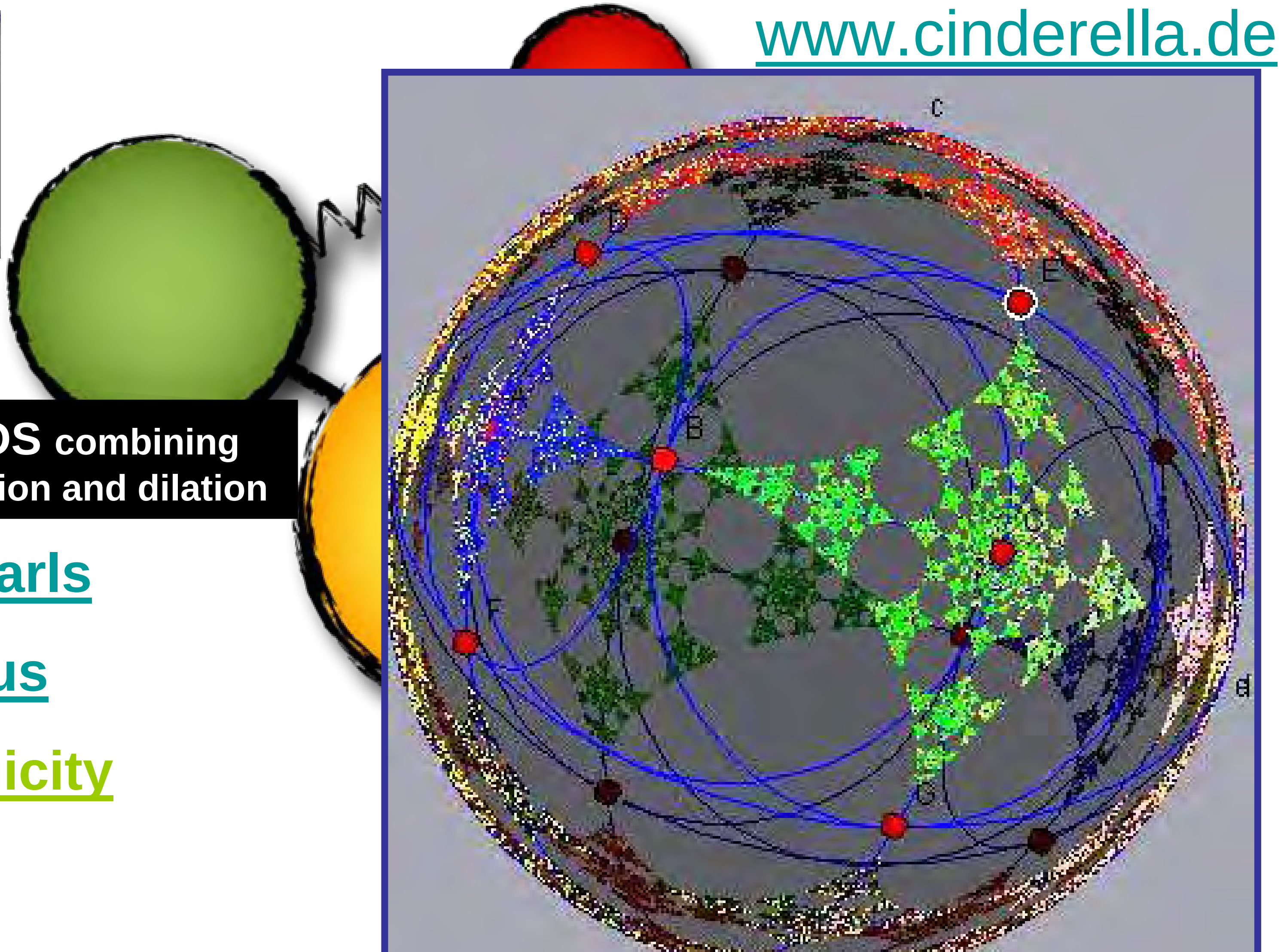
2002: http://klein.math.okstate.edu/IndrasPearls/



CINDERELLA

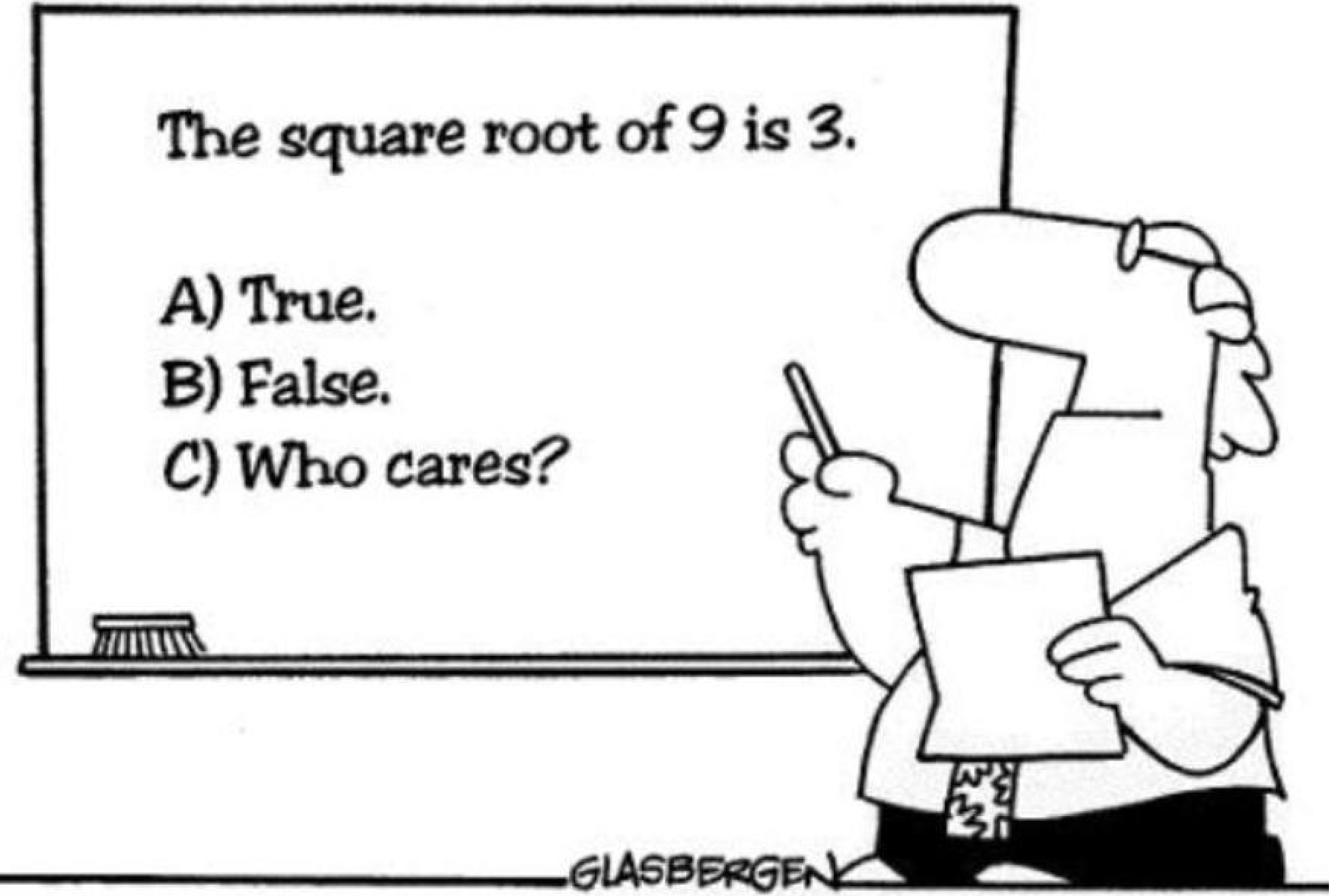


www.cinderella.de



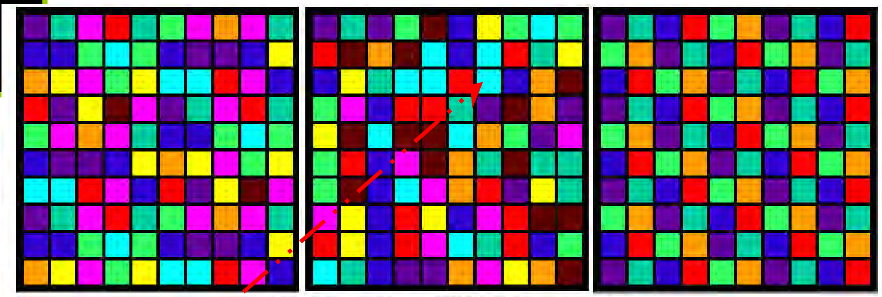
FOUR DEMOS combining inversion, reflection and dilation

1. Indraspearls
2. Apollonius
3. Hyperbolicity
4. Gasket



Many students actually look forward to Mr. Atwadder's math tests.

Inverse & Color Calculators

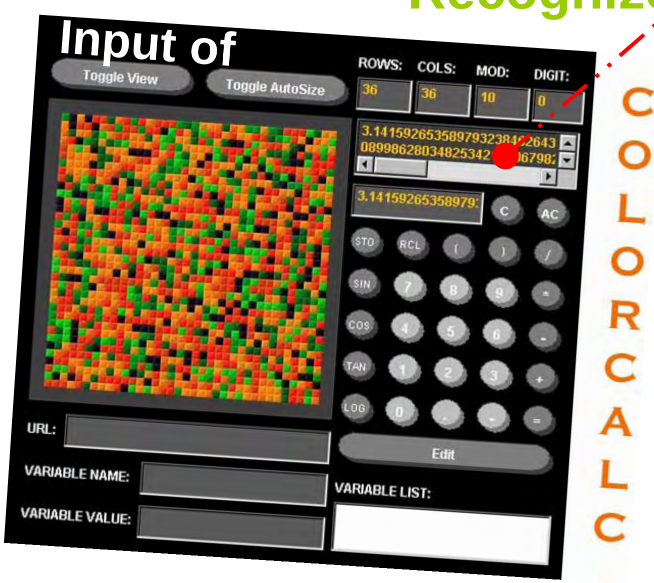


Archimedes: $223/71 < \pi < 22/7$

Inverse Symbolic Computation

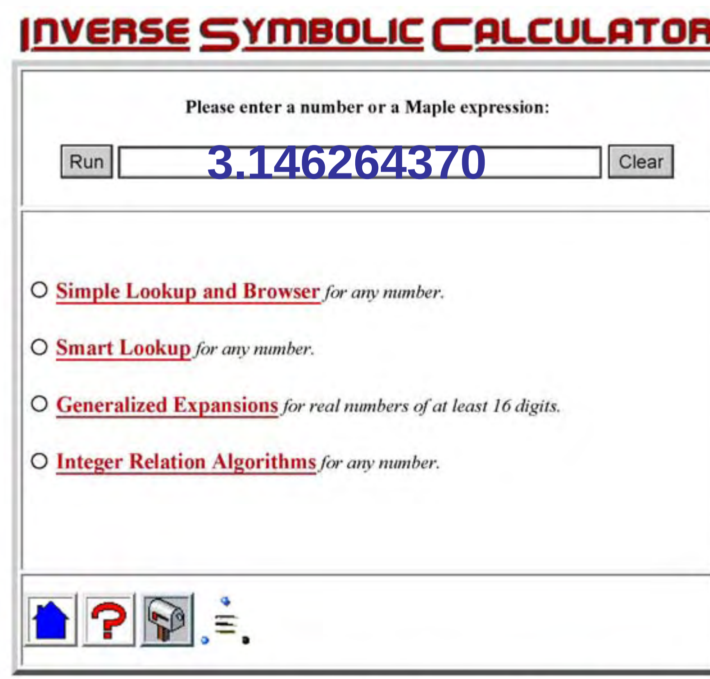
- “**Inferring symbolic structure from numerical data**”
- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs ***micro-parallelism***
- It faces the “curse of exponentiality”

➢ Implemented as **identify** in Maple and **Recognize** in Mathematica



identify(sqrt(2.)+sqrt(3.))

$$\sqrt{2} + \sqrt{3}$$



Expressions that are **not** numeric like ln(Pi*sqrt(2)) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.

ENTERING

evalf(Sum(k^k/k!/exp(k)-1/sqrt(2*Pi*k),k=1..infinity),16)

'Simple Lookup' fails;
'Smart Look up' gives:

INVERSE SYMBOLIC CALCULATOR

The ISC is the **Inverse Symbolic Calculator**, a set of programs and specialized tables of mathematical constants dedicated to the identification of real numbers. It also serves as a way to produce identities with functions and real numbers. It is one of the main ongoing projects at the Centre for Experimental and Constructive Mathematics (CECM).

INVERSE SYMBOL

Results of the search:

Maple output:

.08406950872765600

.8406950872765600e-1

Value to be looked up: .8406950872765600e-1 = K

Performing a smart lookup on .8406950872765600e-1:

Function	Result	Precision	Matches
K-2/3	.5825971579390106666666666	16	1

BOLIC CALCULATOR

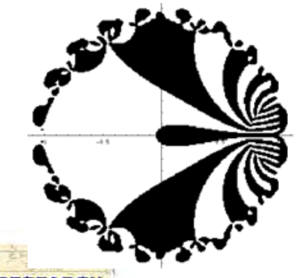
579390106 was probably generated by one
s or found in one of the given tables.
Answers are given from shortest to longest description

Mixed constants with 5 operations
5825971579390106 = Zeta(1/2)/sr(2)/sr(Pi)

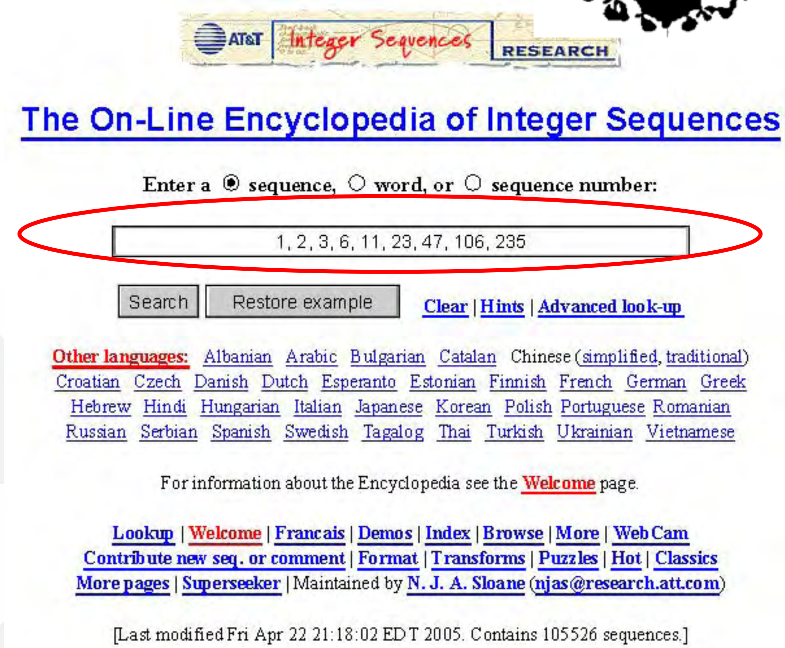
Browse around .5825971579390106.

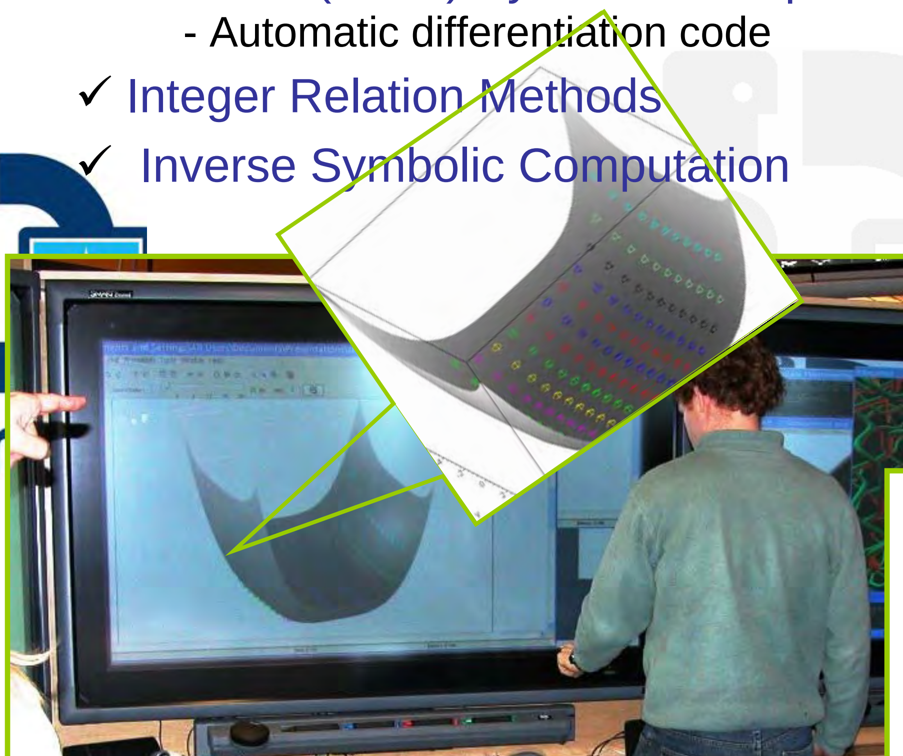

THEORIES
PROOFS
S. Harris

Numeric and Symbolic Computation



- Central to my work - with Dave Bailey - meshed with visualization, randomized checks, many web interfaces and
  - ✓ Massive (serial) Symbolic Computation
    - Automatic differentiation code
  - ✓ Integer Relation Methods
  - ✓ Inverse Symbolic Computation





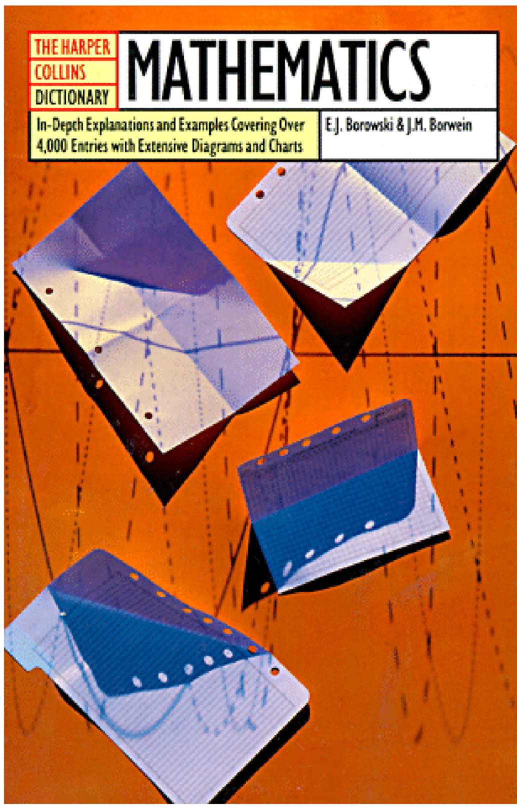
*Parallel derivative free optimization in **Maple***

Other useful tools : Parallel Maple

- Sloane's online sequence database
- Salvy and Zimmerman's generating function package '*gfun*'
- Automatic identity proving: Wilf-Zeilberger method for hypergeometric functions

"Just a darn minute! — Yesterday you said that X equals two!"

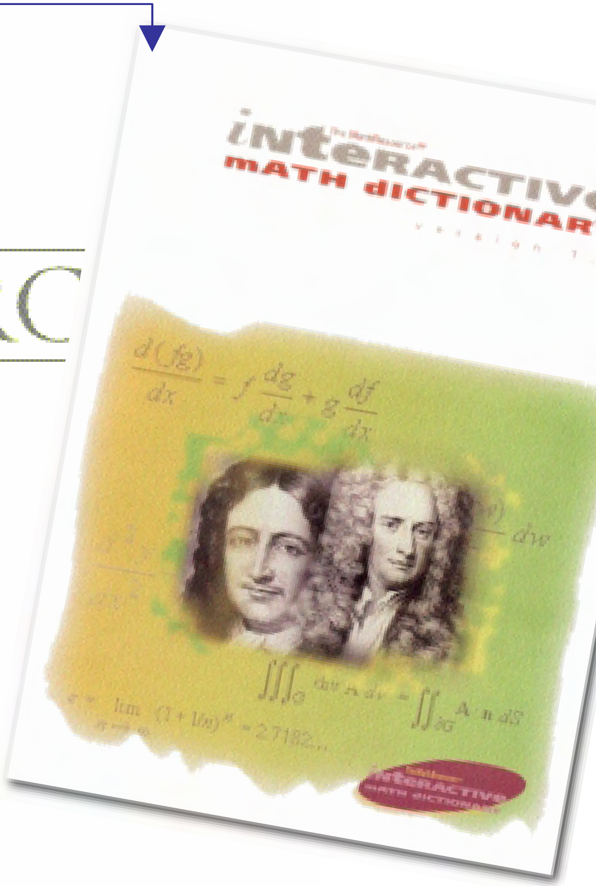
MRI's First Product in Mid-nineties

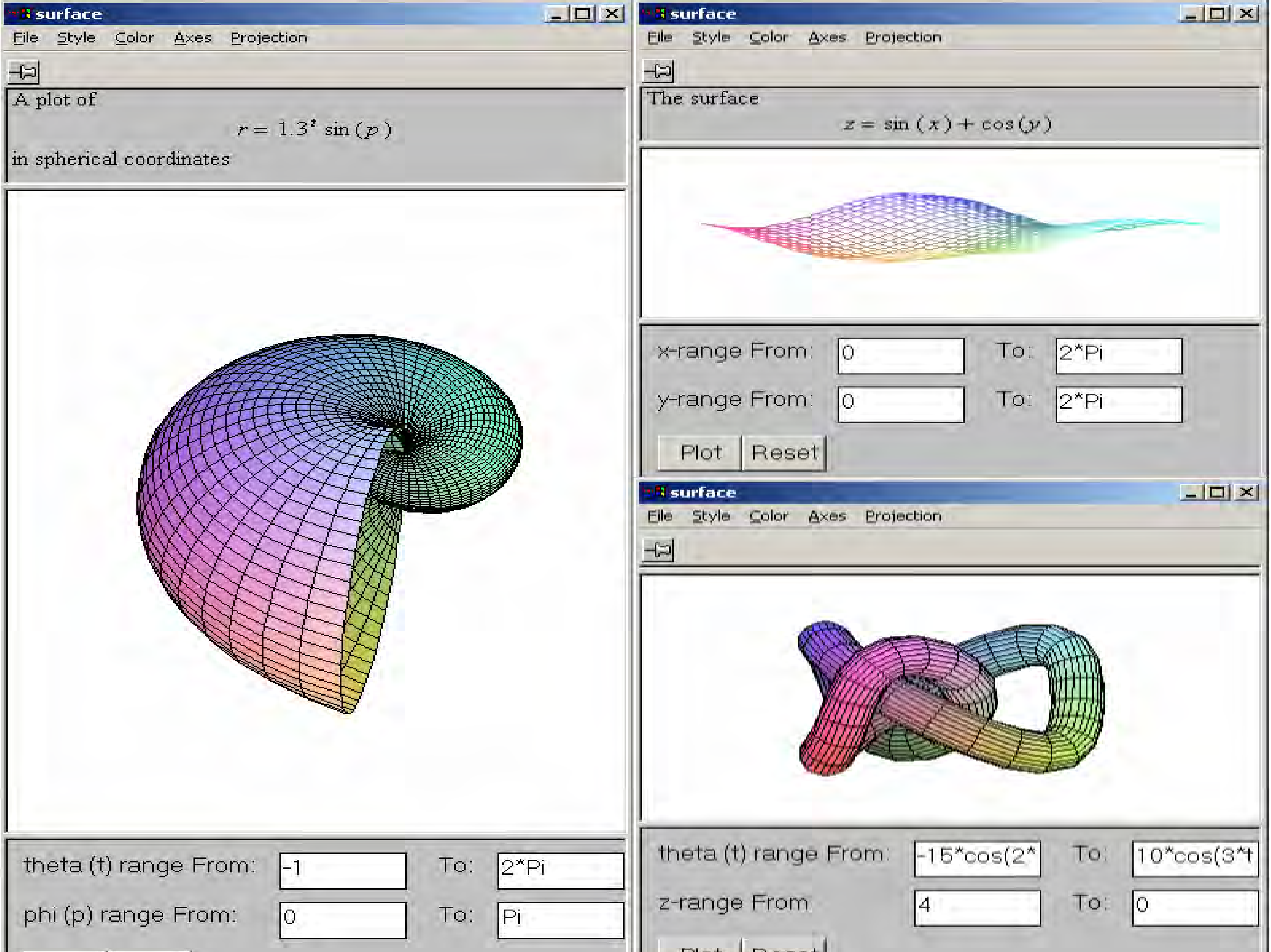


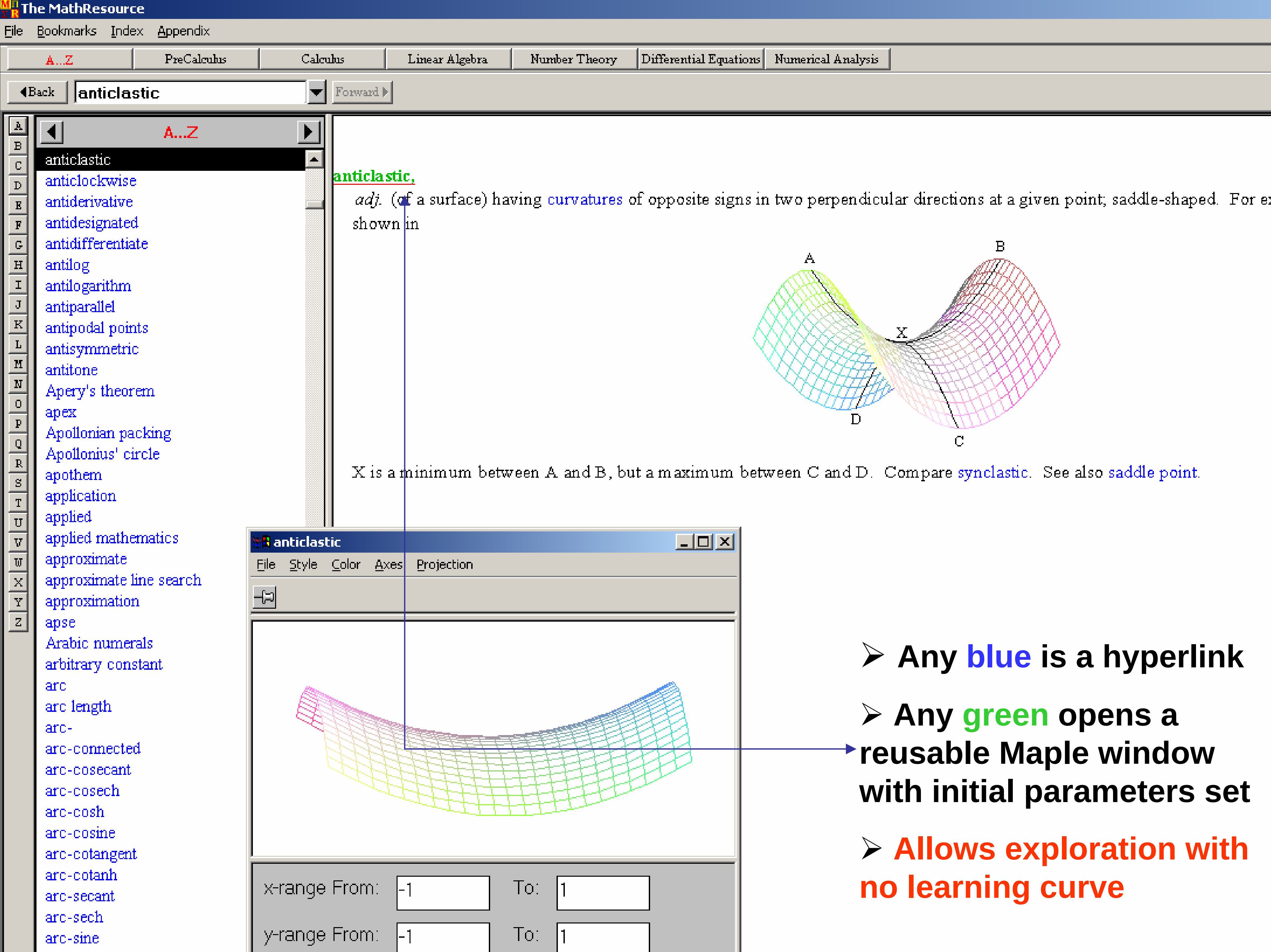
Maplesoft

MathResource

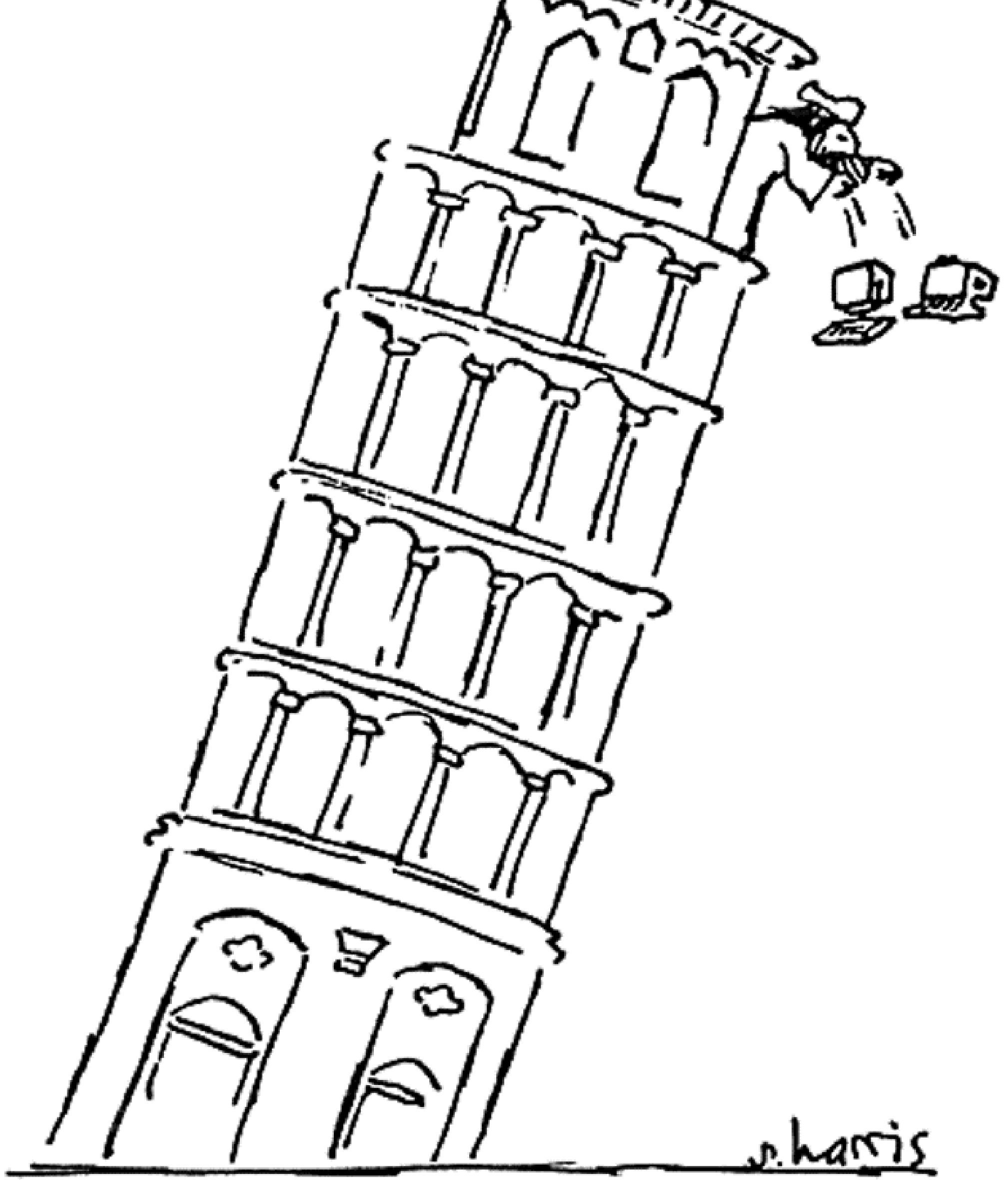
- ►Built on **Harper Collins college dictionary** - an IP adventure!
- ► **Maple** inside the **MathResource**
- ► **Database** now in **Maple 9.5/10**
- ►**CONVERGENCE?**




surface
File Style Color Axes Projection
A plot of
$r = 1.3^t \sin(p)$
in spherical coordinates
theta (t) range From: -1 To: 2*Pi
phi (p) range From: 0 To: Pi
surface
File Style Color Axes Projection
The surface
$z = \sin(x) + \cos(y)$
x-range From: 0 To: 2*Pi
y-range From: 0 To: 2*Pi
Plot Reset
surface
File Style Color Axes Projection
theta (t) range From: -15*cos(2* To: 10*cos(3*t
z-range From 4 To: 0



- Any blue is a hyperlink
- Any green opens a reusable Maple window with initial parameters set
- Allows exploration with no learning curve



IF THERE WERE COMPUTERS IN GALILEO'S TIME

REFERENCES

Dalhousie Distributed Research Institute and Virtual Environment

Enigma

J.M. Borwein and D.H. Bailey, *Mathematics by Experiment: Plausible Reasoning in the 21st Century* A.K. Peters, 2003.

J.M. Borwein, ``The Experimental Mathematician: The Pleasure of Discovery and the Role of Proof,'' *International Journal of Computers for Mathematical Learning*, **10** (2005), 75-108.

D.H. Bailey and J.M Borwein, "Experimental Mathematics: Examples, Methods and Implications," *Notices Amer. Math. Soc.*, **52** No. 5 (2005), 502-514.

"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

• J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.