

- [20] R. WILSON, *Introduction to Graph Theory*, Academic Press, New York, 1972 (4th ed. by Pearson, Harlow, UK, 1996).

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Mathematics by Experiment: Plausible Reasoning in the 21st Century. By Jonathan Borwein and David Bailey. A.K. Peters, Natick, MA, 2004. \$45.00. x+288 pp., hardcover. ISBN 1-56881-211-6.

Experimentation in Mathematics: Computational Paths to Discovery. By Jonathan Borwein, David Bailey, and Roland Girgensohn. A.K. Peters, Natick, MA, 2004. \$49.00. x+357 pp., hardcover. ISBN 1-56881-136-5.

Vladimir Arnold has said that mathematics is the part of physics where experiments are cheap.

Certainly “thought experiments,” which are not unheard of in other parts of physics, are the bread and butter of mathematics. But mathematics also has resorted to physical experiments. Every reader of Courant and Robbins’ *What Is Mathematics?* remembers how the mathematics of minimal surfaces was tested by a physical realization as soap films.

In the books under review, “experimentation” means above all experimentation on computers.

Of course, computer modeling has become pervasive in all science. Every physical, biological, and social phenomenon conceivable is being modeled to death by computers. However, the idea of computer modeling of *mathematics* is still not universally understood. After all, isn’t computation just the same as mathematics?

On the contrary! If we gladly grant that computation is an important, interesting part of mathematics, let’s hope that in return it will be granted that some important, interesting parts of mathematics are not computations. When computers model fluid flow, for example, they are also modeling the behavior of solutions of certain nonlinear partial differential equations. Modeling a hurricane is done by modeling (not solving) a certain mixture of nonlinear hyperbolic, elliptic, and parabolic equations.

Everyone understands that mathematical discovery starts with heuristics, with searching for “patterns”—interesting regularities. Gauss and Lagrange searched the mathematical tables of their day and discovered the prime number theorem—which was not to be proved for another century. Seventy years ago Lehmer used special purpose computers to find interesting patterns about whole numbers. Since Giant Electronic Brains came in 50 years ago, many other mathematicians used computers to explore the integers.

At Simon Fraser University in British Columbia, there is a Centre for Experimental and Constructive Mathematics, founded and led by Jonathan Borwein. It produces the very distinguished and well-established *Journal of Experimental Mathematics*. Now it has also produced these two books, which tell the world what has been and is being accomplished by experimental mathematics. “Our goal in these books is to present a variety of accessible examples of modern mathematics where intelligent computing plays a significant role.” Physical models are virtually absent. The thrill is in discovering and proving interesting new facts about prime numbers, polynomials, identities for special functions, definite integrals, summation of series, combinatorial partitions, And, most deliciously, that fascinating elusive old number called Π .

The first volume is meant to be readable for “anyone with solid undergraduate coursework in mathematics.” Its first and last chapters give historical background and discuss philosophical questions. It is made absolutely clear that experiment is no substitute for proof. But, experiment can sometimes *find the way* to proof. What is made clearest of all is that in designing and carrying out mathematical “experiments”—well-designed explorations—there is a lot to know. Just as in other parts of mathematics, knowing the right tricks, technical mastery, experience, and know-how make all the difference.

“Most” of the second volume’s “several chapters of additional examples. . . should be readable,” in the opinions of the authors, by persons with upper division undergraduate- or graduate-level coursework.

The overall flavor is highly classical. The integers, primes, polynomials, definite integrals, series summation, and formulas about “special functions” gamma, zeta, multizeta, are presented.

There have been some startling discoveries. For example, as a culmination of the incredible history of calculations of Π (now up to *trillions* of digits) there is presented an infinite series for Π that is almost a geometric series:

$$\begin{aligned} \Pi = \sum \text{ from } i = 0 \text{ to} \\ \infty (1/16^i (4/[8i + 1] - 2/[8i + 4] \\ - 1/[8i + 5] - 1/[8i + 6])). \end{aligned}$$

The proof takes only a few lines of elementary calculus.

However, its discovery was a long story, starting with experimental mathematics on the square root of 2 and culminating with the very remarkable “PSLQ” algorithm

Suppose you are interested in a finite collection of real numbers. Is it possible that they are “rationally dependent”? That some linear combination of them, *with integer coefficients*, equals zero?

Many important identities are precisely rational dependences of this kind.

If you suspect there is such a rational dependence, how would you find it effectively, with a reasonable amount of searching, in a reasonable amount of time?

The PSLQ algorithm may do the job. Helaman Ferguson (better known as a mathematical sculptor) invented it in 1993. It was the key tool, for instance, in finding the elementary-looking series for Π written above.

The two books are written in an inviting, conversational, unprepossessing style. They are fascinating as a vast collection of interesting facts, anecdotes, and examples about numbers, primes, polynomials, special functions, definite integrals, series summations, and especially Π . They may even teach interested mathematicians how to become mathematical experimentalists. If a reader wants to dig deeper and follow the argumentation in detail, perhaps with the hope of joining the ranks of mathematical experimentalists, he/she will find that good facility with the standard manipulations of advanced calculus is a must.

The books are supplemented by a useful web site at <http://www.expmath.info> or <http://crd.lbl.gov/~dhbailey/expmath/>.

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Geodynamics of the Lithosphere. By Kurt Stüwe. Springer-Verlag, New York, 2002. \$77.95. xiv+449 pp., hardcover. ISBN 3-540-41726-5.

This book by Kurt Stüwe is a great and interesting introduction to quantitative physical geology for those with the appropriate mathematics and physical science background. It uses the basic principles of heat transfer, energy conservation, hydrostatics, elasticity, etc., and applies them to an enormous variety of geologic phenomena.

The chapter titles are as follows: Chapter 1: Introduction; Chapter 2: Plate Tectonics; Chapter 3: Heat and Temperature; Chapter 4: Elevation and Shape; Chapter 5: Force and Rheology; Chapter 6: Dynamics processes; and Chapter 7: P-T-t-D-Paths. But the book covers far more topics than the number of chapters would suggest.

Chapter 1 introduces a philosophy of physical models and modeling and explains what is necessary for a “good” model as well as the difference between a “good” versus an “accurate model.” The author stresses the point that accurate models are often not practical or needed to understand many phenomena. He also explains the dimensional aspects of geologic modeling (ways and limits of using 1D, 2D, and 3D models) and why his book “refrains” from the complexities of 3D models.

The next chapter introduces plate tectonics. This important theory is explained in full with an overall history and motivation for the theory. Three highly detailed, global maps show the physical evidence that strongly supports plate tectonics such as ocean topography, crustal stress fields, and seismic activity. The maps are so detailed that I wished they were rotated and scaled to each fill a whole page. Other important topics of tectonics are covered such as the structure of the Earth’s lithosphere, plate