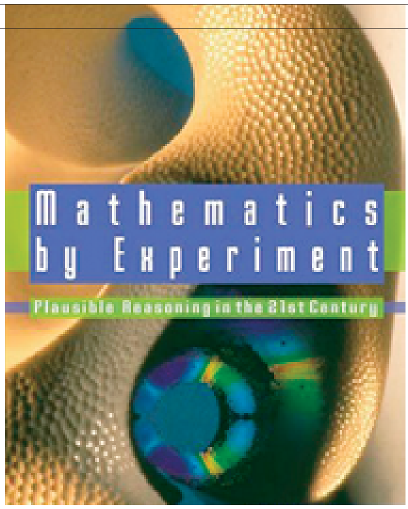
Mathematics by Experiment: Plausible Reasoning in the 21st Century



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By Jonathan Borwein and David Bailey
Reviewed by Padraig Murphy

BISMARCK ONCE SAID that the making of laws, like the making of sausages, should never be watched. Many mathematicians have traditionally thought the same of theorems, as the intuition that suggests a theorem is rarely presented in texts and papers. While rigorous proof has been seen as the *raison d'être* of mathematics, some believe the desire for elegance and terseness in proofs has led to a bewildering style that can obscure underlying motivation. The result is a debate in which leading mathematicians can be found on either side. The Hungarian mathematician George Polya wrote that "rigorous proofs are the hallmark of mathematics; they are an essential part of mathematics' contribution to general culture." But according to Jacques Hadamard, "The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

The debate does not just concern how mathematics is presented, but also how it is done. Mathematicians in the Hadamard school of thought are sympathetic to a more experimental approach that uses computers to generate theorems. In fact, when invited recently by www.edge.org to suggest two rules-of-thumb for mathematics, Professor Steven Strogatz of the Center for Applied Mathematics at Cornell gave as his first law: "When you're trying to prove something, it helps to know it's true." His second was: "To figure out if something is true, check it on the computer. If the machine agrees with your own calculations, you're probably right."

In *Mathematics by Experiment: Plausible Reasoning in the 21st Century*, Jonathan Borwein of Simon Frasier University and David Bailey of Lawrence Berkeley National Laboratory provide an interesting addition to this debate. As the title suggests, the authors side with the experimentalists. Very sensibly, though, they avoid dry philosophical debate, arguing instead for the fruitfulness of the experimental approach in the most enjoyable way possible: through examples of how effective it can be.

The authors point out that the learning-by-calculating method has a distinguished history. They quote 19th century mathematician Johann Gauss, who worked "through systematic experimentation." They also discuss the history of the Riemann hypothesis, one of the most famous unsolved problems in mathematics. While many have assumed that Riemann's conjecture was due to pure insight, his papers show that he performed brute-force calculations to several decimal places.

Nothing handles brute-force calculations like a computer, and with the advent of the computer, the experimental method came into its own. Borwein and Bailey present many examples of how computers can be used to generate theorems. One method uses a computer to evaluate an infinite series to a high number of decimal places, then employs "constant recognition" software to see if this number is approximately equal to a well-known constant, like a fraction times a power of π, or a value of the Riemann zeta function. The resulting conjecture, that the equality holds exactly, can then be proved (one hopes) using more conventional means.

As well as extensive discussions of important problems, there are toothsome mathematical morsels at the end of several chapters. These include quotations from mathematicians, pithy proofs of easily stated theorems, links to interesting Internet sites, and a discussion of the infamous $\pi = 3$ chapter of the Bible. Although often lacking any obvious thematic structure, these sections are among the most enjoyable in the book.

Mathematics by Experiment is not a systematic exposition of any mathematical field, and the reader needs a reasonable knowledge of undergraduate mathematics—especially analysis and number theory—to fully enjoy the book. However, the mathematical ideas are always very good, and the reader is often encouraged to engage them by writing simple computer programs, which affords more pleasure than a passive read. The authors also present ten challenging problems, but one must be prepared to invest a significant amount of time to solve them (or, as the author of this review can attest, *not* to solve them). Whether you take sides in the debate about the validity of experimental mathematics or are not sure what all the fuss is about, the book is a rewarding read.

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