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MR2473161 (Review) 00A35 (37-04)
Borwein, Jonathan; **Bailey, David**
★**Mathematics by experiment.**
Plausible reasoning in the 21st Century.
Second edition.
A K Peters, Ltd., *Wellesley, MA*, 2008. *xii*+377 *pp*. $69.00. *ISBN* 978-1-56881-442-1

It is sometimes claimed, especially in view of the 19th century arithmetization of mathematics, that there is a fundamental asymmetry between the discrete and the continuous in mathematics. "The philosophy of arithmetic is dominated by the foundational approach, while the philosophy of geometry is mainly anti-foundational" [L. Kvasz, *Patterns of change*, Birkhäuser, Basel, 2008; MR2438089 (2009k:00002) (pg. 11)].

The reason for this is simply the fact that one cannot force somebody else to see. Lewis Carroll has beautifully illustrated this fact in his little piece "What the tortoise said to Achilles" (1895). The title alludes to one of Zeno's paradoxes of motion in which Achilles could never overtake the tortoise in a race. In Carroll's dialogue, the tortoise challenges Achilles to use the force of logic to make him accept the conclusion of a simple deductive argument.

But formal proof and logical argumentation became an essential moment of mathematical experience together with division of labor, social communication and growing abstraction. At the same time chance and failure to foresee or predict what might happen and be true also became quite common within a discrete and seemingly disconnected world. Therefore learning by doing and proving by verification seemed to become as unavoidable as the endless debate about "proofs that prove vs. proofs that explain", which has been ongoing since the days of Bolzano.

Now that the computer takes over most of the doing, humans become free to concentrate on seeing. And the authors approvingly quote Berlinski, who has said that the computer "has changed the nature of mathematical experience, suggesting for the first time that mathematics . . . may yet become . . . a place where things are discovered because they are seen" (p. 1).

Seeing and recognition dominated conceptions of mathematics as early as Plato's (see Meno Dialogue). Something different is intended here in this book, where discrete mathematics and synthetic methods totally predominate. Plato meant the seeing and recognition of ideas, not of definite things or facts. Ideas only were pure and were part of what is common to humans living in the very same world and could therefore be securely grasped through one's own mental vision. The spirit here, the ideology of the present volume, is a different one. It sides with the modern trend of arithmetization, expressing itself as the attitude of technology, and the "seeing" intended refers to the perception of the explicit, distinct and definitely determined, as it comes about through computational experimentation.

Two pictures (including a color plate on page 181) of those well-known diagrams that allow one to "see" the irrationality of square root 2 are presented (more beautiful still is the diagram about square root of 5), but the much more rewarding interpretation of them in terms of unending recursive self-similarity based on the geometry of the continuum is not mentioned. It seems somewhat strange to rely on an indirect argument here, where the seeing is so much superior.

By means of other examples (a wealth of examples, in fact) the authors illustrate the fact that seeing and providing an argument, that is, seeing why might be different things in mathematics as in the everyday world. The authors believe that “mathematics is not ultimately about formal proof” (p. 10), yet they do not want to claim that “computations utilized in an experimental approach to mathematics by themselves constitute rigorous proof of claimed results”. And they accordingly “see the computer primarily as a tool to discover mathematical truths and to suggest avenues for formal proof” (p. 7). From the point of view of human understanding—and the authors believe that “mathematics is and will remain a uniquely human undertaking” (p. 41)—there seems, however, little difference between a formal proof and a calculation. And objectively formal proof also becomes marginalized.

“Indeed, one can ask which is more firmly established: (1) a theorem proven at the end of a difficult 100+-page paper, which only a handful other than the author have read, and which relies on dozens of other results by numerous other mathematicians spanning different fields of research; or (2) a conjectured identity . . . for which no rigorous proof is known but which has been numerically verified to 20,000-digit accuracy? More broadly, we observe from our experience working in this field that there appears to be little, if any, correlation between the difficulty of discovering a new relation or other fact and the difficulty of subsequently proving it.” (p. 294)

One cannot find a place in the book, however, where the differences between experimentation in the natural sciences and computational experimentation, or related philosophical issues, are discussed in a more or less systematic manner. The field of computational exploration might bring about a new mathematical culture without many great theories or far reaching results. In the natural sciences people perform experiments in order to find meaningful laws of nature, not to just establish any kind of regularities. In computer mathematics one encounters many regularities which seem true for no reason at all and which have no meaning at all (see for example the very interesting discussion on “normality of numbers” in Chapter 4). In a review of the first edition of the present book this fact was welcomed as a “paradigm-shift”: “Thanks to Its Omnipotence, The Computer, math, that last stronghold of dear Plato, is becoming (overtly!) experimental, a posteriori and even contingent” [D. Zeilberger, Amer. Scientist **93** (2005), no. 2, 182–183)].

The authors themselves describe their aims as follows:

“Our goal in these books is to present a variety of *accessible* examples of modern mathematics where this type of intelligent computing plays a significant role (along with a few examples showing the limitations of computing). We concentrate primarily on examples from analysis and number theory, as this is where we have the most experience, but there are numerous excursions into other areas of mathematics as well. For the most part we content ourselves with outlining reasons and exploring phenomena, leaving a more detailed investigation to the reader. There is, however, a substantial amount of new material, including numerous specific results that as far as we are aware have not yet appeared in the mathematical literature. . . . Most of the first volume should be readable by anyone with solid undergraduate course work in mathematics.” (page VIII/IX)

To such readership the volume offers an overwhelming variety of instructive examples, information and experience. The book at hand is a rich work, written by two experts in the field. The amount of information as well as the manner in which it is presented makes it appear less a book than a laboratory manual or a Web site.

{For a review of the first edition see [*Mathematics by experiment*, A K Peters, Natick, MA, 2004; MR2033012 (2005b:00012)].}

Reviewed by *Michael Otte*

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