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★**Experimentation in mathematics.**

Computational paths to discovery.

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Nowadays it is hard to imagine that some twenty years ago, around 1984, personal computers were not in use very much and most mathematicians relied on pencil and paper for their scientific work. Today virtually every one of us uses internet resources such as email, WWW and TEX for our typesetting. Moreover, there are an increasing number of mathematicians who use the computer as a tool for doing mathematics just as astronomers use telescopes for their work. Of course not all fields of mathematics are amenable to computer treatment. But for the ones that are susceptible, it is very interesting to contemplate what the future will bring. With this in mind the authors of the present book have written a two-volume work, *Experimentation in mathematics*, of which this is the second part. The first part, subtitled 'Plausible reasoning in the 21st century' [J. M. Borwein and D. H. Bailey, *Mathematics by experiment*, A K Peters, Natick, MA, 2004; MR2033012 (2005b:00012)], is written at the undergraduate level, the present volume is an extension to the graduate level.

The point of view of the authors is that mathematical discovery through experimentation and the use of increasingly intelligent software is going to play an indispensible role in mathematics. To support this point of view the authors present a very large variety of mathematical subjects where 'intelligent computing plays a role', as the authors say in the introduction. Although computer calculation does play a role in many parts of the book, it is my impression that the promise of a new approach to mathematical discovery has not been fulfilled, at least, not in the part under review. For example, Chapter 2, which is about Fourier theory, contains a very interesting and entertaining approach to this theory, which was at the basis of much of modern analysis. Of course, in teaching Fourier theory, the use of a computer can release us from much of the drudgery of calculation that the subject entails. The computer can also be used as a tool to explore examples which illustrate the pitfalls that beset Fourier theory. And it can be used to plot the many strange Fourier sums to the point where we can actually 'see' nowhere differentiable continuous functions. However, in this chapter I do not get the impression that a use of the computer has added anything to the theory of Fourier series and transforms. Most of the interesting phenomena were known in the pre-computer era, an exception being perhaps the sinc-integrals.

Even though I believe that a 'computational approach to discovery' has a limited scope within mathematics as a whole, this does not disqualify the book by any means. Being a lover of strange and remarkable mathematical facts and oddities it was a delight to simply browse through the book. There are seven chapters, each ended by a section 'Commentary and additional examples' whose length approximates the length of the preceding chapter most of the time. In these additional sections we find many small facts and problems to be solved from, for example, the American Mathematical Monthly and the well-known 'Berkeley problems'.

The number of topics addressed in the book is enormous and it is futile to give an overview of them. Let me say that I particularly enjoyed reading Chapter 3, which is on the Riemann zeta-function, its values at the positive integers and the related multizeta values. The latter has been the subject of intensive study during the last ten years with links in other fields of mathematics and even in theoretical physics. Of the many sections on numerical methods I would like to mention the Tanh-Sinh quadrature, discovered in the 1970's, which allows extremely fast high precision calculation of many classical constants. This is dealt with in Chapter 7.

Much of the material in the book has arisen from the experiences of the authors while working on a computer based approach to different topics in mathematics. The variety obtained in this way is impressive, the authors have really touched and produced a treasure trove of lovely mathematical gems. Anyone who can appreciate such an attitude to mathematics is bound to enjoy it.

Reviewed by *F. Beukers*