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Number theory

Does Occam's razor shave too close ?

Or is fate such a great provider ?

« I have myself always thought of a mathematician as in the first instance **an observer**, a man who gazes at a distant range of mountains and notes down his observations. »

« **Beauty** is the first test.
There is no permanent place in the world for ugly mathematics. »

Godfrey H. Hardy (1877-1947)

« I have the result, but I do not yet know **how to get it.** »
Carl F. Gauss (1777-1855)

Does Occam's razor shave too close ? Or is fate such a great provider ?

A reminder of

observations presented at the ICM 1998

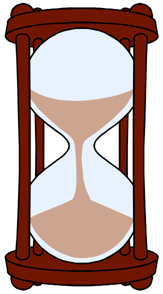
(ad-hoc short com):

1) The fixed point of the function **Ahmes(x) := ((x+1) / x)** $^{(x+1)}$
equals π with a 0.02 % error, and the point at infinity is e.

2) The fixed point of **Rhind(x) := (1 + 1/4 + 1/4² + 1/4³ + 1/ 4**$^{(x+1)}$ **)** ^{4}
approximates π to 0.000 1 % !

3) 3 + 1/8 on 6 bits (i.e. 11.0010), completed by the **first 19 bits of e written from right to left** equals π with a 0.000 001 % error (25 bits in common) ! See ref. [1] and Appendix.

Why are these numbers so close to π ?
And why is there still no explanation 8 years later ?



The aim of this presentation

1) To present remarkable new **observations.**

2) To convince you that it is worth trying **to explain** some of them
with mathematical arguments even when random chance seems enough

Research must not be discouraged by the heuristic maxim called:

Occam's razor principle
(the principle of parsimony)

which says :

Do not introduce unnecessary assumptions or hypotheses in the explanation of a phenomenon.

K eep
I t
S hort and
S imple

English logician and Franciscan
friar William of Ockham (1285-1349?)

Brother William (The Name of the Rose, Umberto Eco)
a Franciscan monk in 1327, inspired by the character of
William of Occam.

OUTLINE

- 1) It is worth **looking for mathematical explanations.**
- 2) Coincidences in the **first bits of π and e.**
- 3) The occurrence of remarkable strings of digits, like **"01234567" in the decimals of 1/81,** can be explained.
- 4) An elementary unexplained observation: the **Champernowne constant** $0.123\ldots1011121\underline{31415}\ldots$
- 5) A dozen of unexplained occurrences starting from:

$$(\pi+2)/(\pi+1) \approx (\log(\pi+1))/\log\pi = 1.2414\,1\ldots$$

$$\text{and} \quad \sqrt{2} = 1.4142\,1\ldots$$

$$\leftarrow$$

- 6) The simplest conclusion is not always the right one: A very **long string of zeros** after the decimal point.

The first 19 bits of e in the sequence of the binary digits of π

e = ...0**10.10110111111000010**...

$\rightarrow$

The first occurrence of these 20 bits written from right to left

is at **position 6 in π !**

π = ...000 11.0010 **0100001111110110101**0 1000...

$\leftarrow$

... ...000 1100000 **0100001111110110101**0 1111...

The second is at position 1 948 205

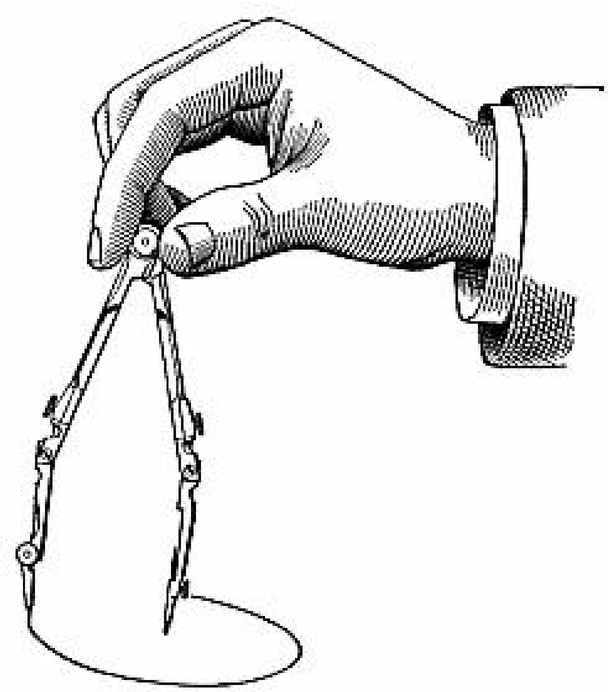
(= almost 2 million).

The length of the circle of radius 1 is:

$$2\pi = 6 + 1/2^2 + \varepsilon_1$$

and $\varepsilon_1 \approx 1/30$

begins with the first 19 bits of e in reverse.



20 bits of e in the first bits of $2\pi - (5/2)^2$ and $2\pi / (5/2)^2$

The period of the exponential function is $2\pi i$ $\qquad e^{2\pi i} = 1$

$2\pi = 25/4 + \varepsilon_1 = (25/4)\ (1 + \varepsilon_2)$

ε_1 has 19+1 bits in common with e (in reverse).

ε_2 has 13 bits in common with e followed by the first 13 bits of 2π (or π):

$$2\pi = \underbrace{\underline{\mathbf{110.010}}}_{25/4}\ \ \underline{\mathbf{0100001}1111101101010}\ \ldots$$

$$\leftarrow \qquad \approx e\ (20 \text{ bits})$$

$$1 + \varepsilon_2 = 1.\underline{0000000101011011111\mathbf{1}}\mathbf{100100100001}\ 0\ \ldots = 2\pi / (25/4)$$

$$\approx e\ (7 \text{ zeroes} + 13 \text{ bits}) \quad \boldsymbol{\approx 2\pi\ (13\ \text{bits})}$$

$$e = \ldots\underline{000000\ 010.101101111\mathbf{1}}000010\ldots$$

$$\rightarrow$$

Looking for a series to explain the 19 bits of e (in reverse) at the 6th position of the binary development of π :

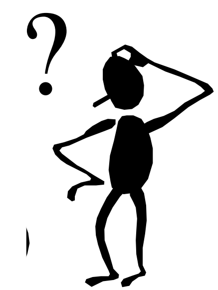
40 bits of π in the first digits of a number close to e

$$e_{\pi 40b} = e - e / 2^{20} + e^2 / 2^{30} - 1 / 2^{31} =$$

$$\underline{010.1011011111100001001001100000000\mathbf{0}000000}\ 1\ 0000\ldots_2$$

$\leftarrow$ $\leftarrow$ π $\leftarrow$ $\leftarrow$

<u>14</u> zeros followed by the first 26 bits of π



This number in base 10 (recreational mathematics):

$e_{\pi 40b}$ = 2.<u>718</u>2 79 <u>2425</u> 19 <u>**4 1 41**42</u>... ("<u>1414</u>" at position <u>14</u>)

$\sqrt{2}$ = <u>**1.41**42</u>… (5 digits in common)

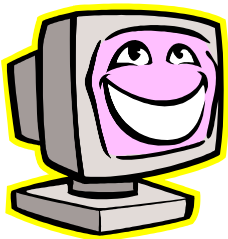
C_b = 0.123...31415 161<u>718</u> ... <u>2425</u> ... 4 <u>0 4142</u>... (The Champernowne constant)

$\sqrt{2} - 1$ = <u>0.4142</u>

Some coincidences do have mathematical explanations

In base b=10, the number $b/(b-1)^2$ =

$$1 / 8.1 \; = \; 0.\underline{1234567}9\; 0\underline{1234567}9\; 0\ldots \; = \; 0 + 1/ (8 + 1/ (9 + 1/1)) \; = \; [0; 8, 9, 1]$$



has the same 7 first digits that the normal and transcendental **Champernowne constant**:

$$C_{10} \; = \; 0.\underline{1234567}89\; 10\; 11\; 12\; 13\; 14\; 15\ldots_{10} \; \approx \; C_{10,\, cf17}$$

$$C_{10,\, cf17} \; = \; [0; 8, 9, 1, 149083, 1, 1, 1, 4, 1, 1, 1, 3, 4, 1, 1, 1, 15]$$

These 7 digits match because the decimals of $b/(b-1)^2$
begin with the first b-3 digits of the base of numeration.

In the same way, in base b, the number
$b^2/(b^2-1)^2$ is written: $\underline{00}.01\; 02\; 03 \ldots 0(b-2)\; 0(b-1)\; 10\; 11\; 12\; 13\; 14\; 15 \ldots (b^2-3)\; (b^2-1)\; \underline{00}\; 01\; 02 \ldots_b$

It has $2(b^2-b-1)$ digits in common with the Champernowne number at position b-2.
So, in base b=10, 1 / 98.01 has 178 digits in common with C_{10} at position 8 (see appendix).

Some coincidences do not (yet ?) have a mathematical explanation

In every base $b > 8$, the **Champernowne constant** begins in the same way:

$$C_b \;=\; 0\,.\,1\,2\,3\,4\,5\,6\,7\,8 \ldots 10 \quad 11 \quad 121 \quad \underline{3\,\mathbf{1415}\quad \mathbf{1}\;\;\mathbf{6}\;\;1\;\;7\;\;1\;\;8}\;\ldots_b$$

$$\text{b digits of base b} \qquad b \quad b+1 \quad (b+1)^2 \qquad \approx\pi_{10} \qquad \approx\boldsymbol{\varphi}_{10} \qquad \approx(e-1)_{10}$$

$$\approx\sqrt{\mathbf{2}}_{10}$$

$1.415\ldots = [1; 2, 2, 2, \ldots]$

$1.617\ldots = [1; 1, 1, 1, 1, 1, 1, \ldots]$ The golden ratio in base 10 is $\varphi_{10} = 1.618\,0 \ldots$

$1.718\ldots = [1; 1, 2, 1, 1, 4, 1, 1, \ldots]$

Recreational:

In base 10, "3 1415 161 718" occurs between **2 remarkable positions:**

position $16 = 2^4 = 2^{2^2} = \lfloor 16.18\ldots \rfloor = \lfloor 10\,\varphi \rfloor$

and position $27 = 3^3 = \lfloor 27.18\ldots \rfloor = \lfloor 10\,e \rfloor$

The notation $\lfloor \ldots \rfloor$ means the integer value.

6 independent remarkable approximated formulas waiting for an explanation

1

$\underline{01.241}4...\quad = \quad(\pi + 2) / (\pi + 1) \quad\approx\quad (\log(\pi + 1)) / \log\pi \quad = \quad 0\underline{1.24141}...$

← ←

5 digits in common with 3

$\log(\pi + 1) \;=\; \underline{1.4210}...$

→

and with $\sqrt{2}$ or $\sqrt{2} - 1$ 2

6 digits in common with 2

$\sqrt{2} \quad = \quad 0\ \underline{1.4142\ 1}...$

→

$1 / (\pi + 1) \quad = \quad 0.\underline{2414}...$

↔ 5

$\sqrt{2} + 1 \quad = \quad \underline{2.4142}...$

↔

$\pi + 2 \quad = \quad \underline{5.1415}...$

↔

$\pi + 1 \quad = \quad \underline{4.141}...$ 4

←

$\approx \sqrt{2}$

$\sqrt{2} - 1 \quad = \quad 0\underline{.4142}...$ 5

←

6

$$(\underline{4.1410}414 \;/\; 3.14104\underline{141421})^{\underline{4.14104}\underline{141421}} = 3.1410414...$$

← → ← → $\approx \pi$

A 7^{th} and 8^{th} coincidence related to $(\pi + 2) / (\pi + 1) \approx (\log(\pi + 1)) / \log\pi$

$$Ah(x) := \left((x+1)/x \right)^{(x+1)} \xrightarrow[x \to \infty]{} e \quad \text{and} \quad Ah^{\infty} \approx \pi$$

$Ah^{\infty} = 3.141\,041\,525\,410\ldots$ is the solution of the equations:

$Ah(x) = x$ (the fixed point of the Ahmes function) i.e.

$(x+2) / (x+1) = (\log(x+1)) / \log x$ i.e.
$(x+2) / \log(x+1) = (x+1) / \log x$ but

$$(x+2) / \log(x+1) \underset{x \to \infty}{\sim} (x+1) / \log x \underset{x \to \infty}{\sim} \pi(x) \quad \text{(the prime number theorem)}$$

The deviation at $x = Ah^{\infty}$ is:

$(Ah^{\infty} + 2) / \log (Ah^{\infty} + 1) - \pi(Ah^{\infty}) =$
$\mathbf{(Ah^{\infty} + 1) / \log Ah^{\infty} - \pi(Ah^{\infty}) = 1.61803}\,8\ldots \approx$
$\boldsymbol{\varphi} = \mathbf{1.61803}\,3\ldots$ (6 digits in common !)

A 9^{th} and 10^{th} coincidence related to

$(\pi + 2) / (\pi + 1) \approx (\log(\pi + 1)) / \log\pi$

log (π+1) = $\underline{1.4210}$ ... has **5 digits in common** (in reverse) with

(log(π + 1)) / log π = $\underline{\mathbf{01.241}}\mathbf{4}$ 1 ... but

$\leftarrow$

log(e+1) = 1.3 13 ... has **all its digits in common with**

(log(e+1)) / log e and more:

(e+2) / (e+1) = 1.2689 $\underline{\mathbf{41421}}$ $\underline{3}$6 999 ... **has 5 digits in common** with

$\rightarrow$

(log(π + 1)) / log π and (π + 2) / (π + 1) = 0**1.2414**... (the bold ones)

and 6 (almost 7) with √2 = 1.$\underline{\mathbf{41421}}$ $\underline{3}$56 ... ≈ 1.$\underline{\mathbf{41421}\ 3}$6

3 more coincidences related to $(\pi + 2) / (\pi + 1) \approx (\log(\pi + 1)) / \log\pi$

$(\pi + 2) / (\pi + 1) = \underline{\mathbf{1.2414}}\ldots$ has 5 digits in common (in reverse) with

$\leftarrow$

$\sqrt{2} = 1.\underline{\mathbf{41421}}\ldots$ (and $(\log(\pi + 1)) / \log\pi = \underline{\mathbf{1.2414}}\,1\ldots$) but

$\leftarrow$ **$\approx\sqrt{2}$ (6 digits)**

$(\sqrt{2} + 2) / (\sqrt{2} + 1) = \sqrt{2}$ has all its digits in common with $\sqrt{2}$.

$(e+2) / (e+1) = 1.2689\,\underline{\mathbf{41421}\,3}6\ldots$ has 6 (and almost 7) digits in common with

$\rightarrow$

$\sqrt{2} = 1.\underline{\mathbf{41421}\,3}56\ldots \approx 1.\underline{\mathbf{41421}\,3}6$

$(1/\varphi + 2) / (1/\varphi + 1) = 1/\varphi + 1 = \varphi$ $= 1.6180339\ldots$

$(Ah^{\infty} + 2) / (Ah^{\infty} + 1) = \underline{\mathbf{1.2414}}8\underline{5141}8088\ldots = (\log(Ah^{\infty} + 1)) / \log(Ah^{\infty})$

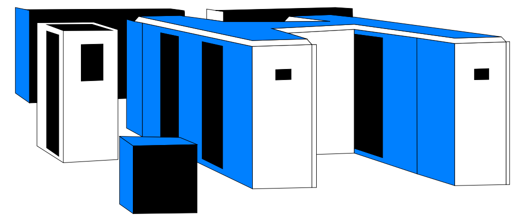
$\approx\sqrt{2} - 1$ $\leftarrow$ $\leftarrow$ (4 and almost 5 digits of $\pi = 3.\underline{1415}9\ldots$)

(The first 3 and almost 4 digits of $\sqrt{2} = 1.41\ldots$)

The simplest hypothesis is not always the right one

We defined $C_{10,\,cf17} = 120.999\,998\,998 \;/\; 980.1 \approx 1 / 8.1$

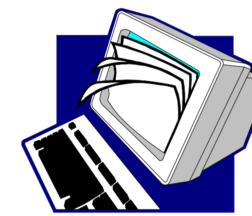
Let $\mathbf{v_0} = \mathbf{1 + C_{10,\,cf17} - C_{10}}$



and $\mathbf{v_1} = \mathbf{8.1 \sum_{k>0} \lfloor k \;\; v_0 \rfloor / 10^k}$

So, $v_1 = 1.0000000000...000...$

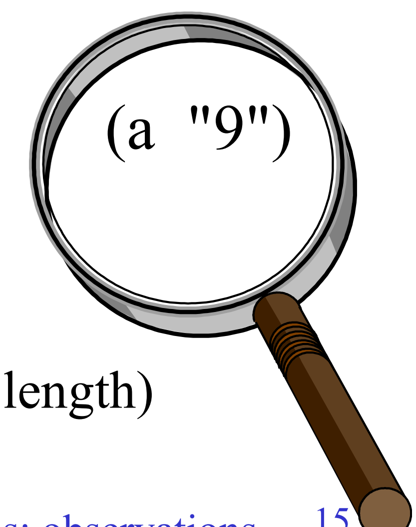
and **the computer prints as many zeros as it can !**

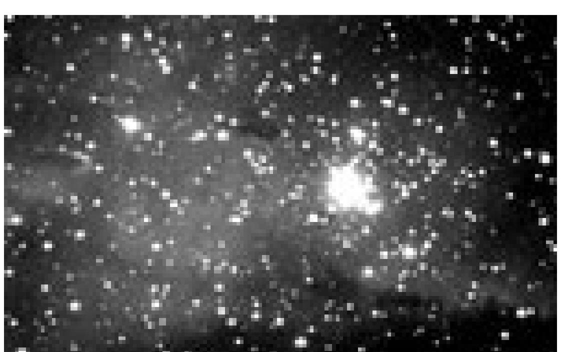


Is this number an integer ? No !

But it has **more consecutive zeros** after the decimal point **than there are Planck cubes** (with edges of Planck length $\approx 1.616 \times 10^{-35}$ m) **in the observable universe** (of radius: 13.7 billions of light years $\approx$ $13.7 \times 10^9 \times 3 \times 10^8$ m/s $\times$ $3.15... \times 10^7$ s), i.e. more than $\mathbf{10^{183}}$.

And do not miss the observation of the first non zero decimal because it is followed by another pack of more than 10^{183} zeros.





(A proton is approximately 10^{-15} m >> Planck length)

A quickly decreasing sequence towards 1

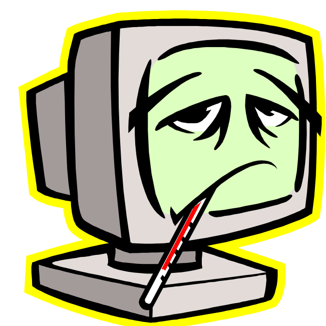
$v_1 \approx 1$ Almost true identities: "**High-precision frauds**" as J. Borwein and D. Bailey call them.

$$\mathbf{v_{n+1} = 8.1 \sum_{k>0} \lfloor k\ v_n \rfloor / 10^k}$$

The second element of this sequence,

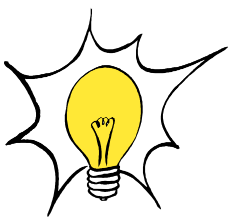
$v_2 = 1.000000000000...$ has more than $10^{10^{183}}$ consecutive zeros after the first "1".

The third element ...

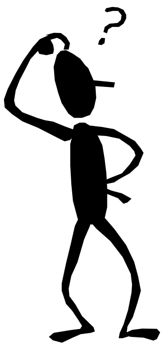


We constructed a decreasing (lower bounded) sequence that converges towards 1 extremely quickly.
(similar ideas are found in ref. [3]).

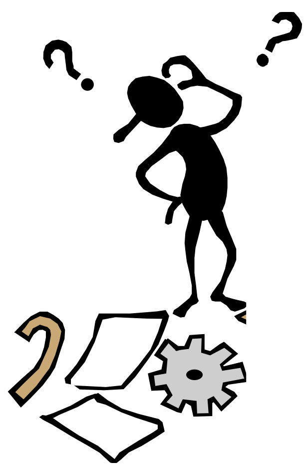
CONCLUSION

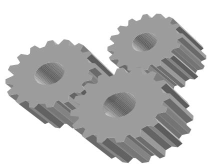


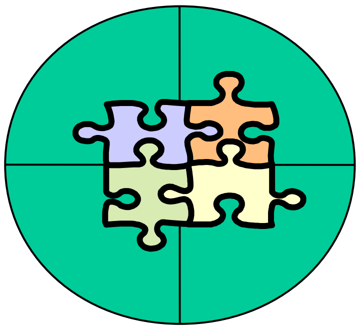
The most exciting phrase to hear in science,
the one that heralds new discoveries,
is not « Eureka! » (I found it!) but
« That's funny... »



Isaac Asimov
(1920-1992)







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[2] : Jonathan M. Borwein, David H. Bailey,
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[3] : Jonathan M. Borwein, David H. Bailey, Roland Girgensohn,
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[4] : Jean-Paul Delahaye, *Le fascinant nombre π,*
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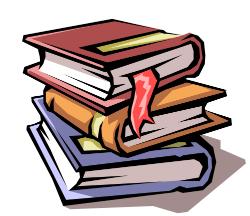

WEB SITES (1)

http://www.expmath.info	:	Experimental Mathematics (with a list of websites)
http://pari.math.u-bordeaux.fr	:	PARI/GP software
http://www.cecm.sfu.ca/projects/ISC	:	Centre for Experimental and Constructive Mathematics
http://oldweb.cecm.sfu.ca/projects/ISC	:	Inverse Symbolic Calculator

WEB SITES (2)

http://pi.nersc.gov or pisearch.lbl.gov :
Search the 4 billion bits of Pi for a string

http://www.research.att.com/~njas/sequences :
The On-Line Encyclopedia of Integer Sequences

http://mathworld.wolfram.com/AlmostInteger.html :
Eric Weisstein's World of Mathematics

CREDITS THANKS TO

CREATIS (**CNRS** UMR 5515, INSERM U 630, INSA, Lyon),
Centre de Recherche et d'Applications en Traitement de l'Image et du Signal,
Research and Applications Center in Image and Signal Processing,
Isabelle MAGNIN, Director, and Pierre SEILLE who used
the technical specifications of the author of these slides to write the code of
the **DigITallOccurrences** software **(**to count and locate strings of characters
or digits in large texts or numbers in any base of numeration).

Laboratoire A2X (**CNRS** UMR 9936, Université Bordeaux I) for the
powerful **PARI/GP** free calculator software (GNU General Public License),
Karim BELABAS, Henri COHEN, etc...

SNECMA (SAFRAN Group, Paris), Information Systems Department.

SMF (Société Mathématique de France),
AMS (American Mathematical Society),

Marie VANOLI.

If you have

explanations for these observations,
remarks or **questions** about this presentation,
information, or
analogous observations to communicate,

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APPENDIX

Occurrences and

Additional observations

A better precision

Previous presentations

Occurrences of 19 bits of e in π and reciprocally

The first occurrence of the digital reflection of the first 19 bits of e in the sequence of the binary digits of π
is at **position 6.**

$\pi =$ ...000 11.00 10 **0100001111110110101** 01000...
...0 1110 110 10 0 **0100001111110110101** 1...

The second is at position 746 913,

the third at position 1 948 205 and the fourth at position 2 388 593.

The first occurrences of the first 19 bits of e are at position 446 009, 758 111, 860 841 and 1 117 446 of the binary development of π.

The first occurrence of the first 19 bits of π is at position 568 313 in the bits of e, and in reverse, at position 685 719.

Other occurrences of "314"
(recreational)

The first non trivial zero of the Riemann zeta function $\zeta(s) := \sum_{k>0} 1/k^s$

is $1 / 2 + \underline{14.13}\ldots\ i$

$\leftarrow \approx \pi$

The position of the zeroes of ζ is used in the demonstration of the prime number theorem (see slide 12).

$\approx -i^2 / 1.414\ldots^2 + 14.14\ldots\ i$

$= -1 / (\sqrt{(-2)})^2 + 10\sqrt{(-2)}$

$\pi = \underline{3.141}592653589\ldots$

$e^\pi \approx 2\,\underbrace{\underline{3.14}}_{}\,\underbrace{\underline{0693}}_{\approx \log 2} \approx 2\,\underbrace{\underline{3.141}}_{\approx \pi}$

$e^\pi = \pi + 10 + \underbrace{\underline{9.999}0\underline{999}7\underline{9}18\underline{9}\ldots}_{\approx 10}$ (Nine "9", and $18 = 2 \times 9$) (see ref. [1])

$\log 2 = 0.69\,\underbrace{\underline{314}}_{\approx \pi}\,\underbrace{\underline{718}}_{\approx e-2 = 0.718\,28\,18\,28\,4\ldots}\,0\ldots = 0.69\,\underbrace{\underline{3141}}_{\approx \pi} + 0.\underbrace{\underline{0000\,0618\,0}}_{\approx 1/\varphi}\ldots$

The simplest hypothesis is not always the right one: the first digits of π with exceptions

A truncated Gregory's series for π :

$$4 \sum_{0 < 2k < 10\,000\,001} (-1)^{k-1} / (2k-1) =$$

$$\underline{3.141592}\ 4\ \underline{5358979323846}\ 4\ \underline{643383279502} \ldots$$

These 32 underlined digits are similar to that of π at the same positions.

But do not infer that the two not underlined "4" are digits of π either:

The digit at position 7 in the decimal development of π is a "6",

the digit at position $21 = 3\times7$ is a "2".

See more in ref. [1] and [4].

Precisions (1)

In base b=10, the rational number $b^2/(b^2-1)^2 = 100 / 99^2 = 1 / 98.01 =$
$\underline{00}.01\ 02\ 03\ ...\ 08\ 0\underline{9\ 10\ 11\ ...\ 97\ 9}9\ \underline{00}\ 01\ 02\ 03\ ...$

has 178 digits in common with the Champernowne constant at position 8 :
$C_{10} = 0.123...8\ \underline{9\ 10\ 11\ ...\ 97\ 9}8\ 99\ 100\ 101\ 102\ \ldots$
and all its digits except the first ones are those of the rational number
$C_{10,\ cf17} = 0.123...8\ \underline{9\ 10\ 11\ ...\ 97\ 9}9\ 00\ 01\ 02\ 03...08\ 0\underline{9\ 10\ 11\ ...}$

$= 604.\underline{99999}499 / 4900.5 = (1 - 10^{-8} \times 1.002 / 1.21) / 8.1$
$= 10 / 9^2 - 10^{-6} \times 10.02 / 99^2$
which has its 185 first digits in common with C_{10}:
$C_{10,\ cf17} - C_{10} = 0.\underline{9101019}... \times 10^{01\text{-}0190}$

$\leftrightarrow$ $\rightarrow$ (recreational mathematics)

$e_{\pi 40b} = 2.\underline{718}2\ 79\ \underline{2425}\ 19\ \underline{4\ 1\ 4142}\ 875...$
$\underline{2.4142}1\ 3... = [2; 2, 2, 2, 2, 2, 2, 2, ...] = \sqrt{2} + 1$

Precisions (2)

$1.4151... = [1; 2, 2, 2, 4, 5, ...]$
$1.6171... = [1; 1, 1, 1, 1, 1, 1, 2, 1, 2, ...]$
$1.7181... = [1; 1, 2, 1, 1, 4, 1, 1, 1, ...]$

(extracts of the Champernowne number in base 10)

$10 \;/\; (\pi + 1) \quad = \quad \underline{2.41}453... = \quad [2; 2, 2, 2, 2, 1, 5, 117, ...] \;\approx\; \sqrt{2} + 1$

$(\pi + 2) \;/\; (\pi + 1) \quad = \quad \underline{01.241}4\ 53...$

$\log(\pi + 1) \;=\; \underline{1.4210}\ 80412...$

$$\underline{1.41421}... \;=\; \sqrt{2}$$

$(\log(\pi + 1)) \;/\; \log\pi \quad = \quad 0\underline{1.24141}\ 11\ \underline{227120}\ 08\ 00286\ 48897\ 70515\ 67002\ 12014$
$58888\ 36628\ 21338\ \underline{227120}\ 43\ldots$

$\log\pi \quad = \underset{\rightarrow}{\underline{1421.0}}\ ...\ /\ \underset{\leftarrow}{\underline{0\ 1241.}}\ ... = \quad 1.14...$

Other interesting values of (x+2)/(x+1) and (log(x+1))/log(x)

There are many other interesting values, for example when
$x \to +\infty$, $x \to 0$, $x \to +1$,

or $x = 2$, $x = \varphi$:

$(2 + 2)/(2 + 1) = 4/3 = 1.33333... \approx 1.33133... = \pi^{1/4}$ (Scribe Ahmes, 1650 bc)

$(0 + 2) / (0 + 1) = (\log(\varphi + 1)) / \log \varphi = 2 = (\sqrt{2})^2$

…

ICM 1998, Berlin, Aug.18-27Abstract of the author's short communication and poster

www.mathematik.uni-bielefeld.de/icm98/abstracts/ps/late_ps.html

Similarities in digit sequences in π and e

Correspondances entre des séquences de chiffres de π et e

Coincidences exist between certain of the digit sequences of universal numbers. Are the corresponding approaching likelinesses due to pure chance or do they stem from an intelligible structure? Examples:

in binary numeration, π bits from the 7^{th} to the 25^{th}, read from right to left, correspond to the first 19 bits of e.

For the function Ahmespb50(x) $= ((x+1)/x)^{(x+1)}$ whose limit is e when x tends towards infinity, the fixed point is

3.1410... = π / 1.00017… The fixed point of the function Rhindpb50(x) $= (1 + 1/4 + 1/4^2 + 1/4^3 + 1/4^{(x+1)})^4$ is

3.1415898... = π / 1.000 000 88...

A natural parameterization of the graph of this function leads to a point of abscissa 3.14159294... = 1.000 000 093... π

"Why so close to π ?"

A partir du problème 50 du papyrus Rhind copié par le scribe Ahmès 1650 ans avant J.C. , voici une approximation de π datant de 1850 ans avant notre ère :

Ahmès pb50(3) $= (4/3)^4 = 3{,}16\ldots$

Ahmèspb50(infini) $= e$ proche de $2\sqrt{2}$ et de π

Le point fixe de la fonction Ahmèspb50(x) = ((x+1)/x) $^{\mathbf{(x+1)}}$ est la racine de l'équation $(x+1)^{(x+1)} = x^{(x+2)}$ et vaut 3,141 0 41 ... c'est-à-dire π à 1 / 5000 près :

$- 1 + 4{,}141\ 0\ 41 \ldots = (4{,}141\ 041 \ldots / 3{,}141\ 041 \ldots)^{4{,}141\ 041 \ldots}$

$\sqrt{2} = 0\ 1{,}414 \ldots$ et $n^2 + n + 41$ est premier pour $n = -40$ à $40-1$

Ahmès pb50(π) $= ((\pi+1)/\pi)^{(\pi+1)} =$ environ $3{,}141$

$((\pi+1)/\pi)^{(\pi+1+1/(2^9 e))} = 3{,}1415926\ 1..$ et $\pi = 3{,}1415926\ 5\ldots$

$1 + 1/(\pi+1+1/(2^9 e)) = 1{,}24141\ 112\ 5\ldots$

$\log(\pi+1) / \log \pi = 1{,}24141\ 112\ 2\ldots = \log 4{,}141\ldots / \log 3{,}141\ldots$

et, écrits de droite à gauche, ...124141 sont les 6 premiers chiffres de $\sqrt{2}$

Le point fixe de la fonction

$$\mathbf{Rhindpb50(x) = (1 + 1/4 + 1/4^2 + 1/4^3 + 1/4^{(x+1)})^4}$$

vaut 3,1415898... = π / 1,000 000 88...

Posons $a(x) = 4 \operatorname{Arctan} (\mathrm{Rhindpb50}(x) / x)$ $x > 0$

Alors, le minimum (avec dérivée nulle) de la fonction

$$F(x) = (x - a(x)) (\mathrm{Rhindpb50}(x) - a(x))$$

vaut 3,1415929... = π / 1.000 000 09...

3+1/8 sur 6 bits, suivis par les **19 premiers bits de e écrits de droite à gauche**

s'écrit 11,0010 **0100001111110110101** 00000 et, en binaire

π = 11,0010 **0100001111110110101** 01000...

(coïncidence sur 6+19 = 25 bits)

Plus précisément :

e = 010,1011011111100010 101 0001011000 1...

$e / 2^{20}$ = 000,0000000000000000 010,1011011...

et la différence entre les bits écrits ci-dessus s'écrit avec 33 bits de π de droite à gauche : 010,1011011111100010 010 0110000000 1

car π s'écrit 00000011,0 010 0100001111110110101010...

Autre observation récréative :

Les 19 premiers bits de e sont positionnés entre les positions 25 et 6 des premiers bits de π . Or les premiers bits de π correspondent aux valeurs 6 = 110,0 en base 2, 25 = 11001,00 en base 2, et 19 = 010011 en base 2, où l'on reconnaît les 6 premiers bits de π de droite à gauche.